UNIVERSITY OF ESWATINI

DECEMBER 2018 MAIN EXAMINATION

IDE B.Com II,III, B.Ed (Sec.) II,III

Title of Paper : Quantitative Techniques

Course Number : MS202/MAT202

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Find the second order derivatives f_{xx}, f_{xy} and f_{yy} for the function

$$f(x,y) = x^3 - 3x^2y + 3xy^2 + y^2.$$

[3 Marks]

(b) The total weekly revenue (in Rands) of Country Workshop associated with manufacturing and selling their rolltop desks is given by the function

$$R(x,y) = -0.2x^2 - 0.25y^2 - 0.2xy + 200x + 160y$$

where x denotes the number of finished units and y denotes the number of unfinished units manufactured and sold each week. Compute $\dfrac{\partial R}{\partial x}$ and $\dfrac{\partial R}{\partial y}$ when $x = 300$ and $y = 250$. Interpret your results. [5 Marks]

(c) Tee and Jay carpenters manufactures both finished and unfinished furniture for the home. The estimated quantities demanded each week for its rolltop desks in the finished and unfinished versions are x and y units where the corresponding unit prices are

$$p = 200 - \frac{1}{5}x - \frac{1}{10}y$$
$$q = 160 - \frac{1}{10}x - \frac{1}{4}y$$

Emalangeni, respectively. Find the weekly total revenue function $R(x,y)$ [3 Marks]

(d) Malkerns research centre recommends that farmers must spread out at least 5000 kg of phosphate fertilizer and not less than 7000 kg of nitrogen fertilizer to raise the productivity of their crops on farms. There are two mixtures A and B, weighing 100 kg each, from which these fertilizers can be obtained. The costs of mixture A and B are R40 and R25, respectively. Mixture A contains 40 kg of phosphate and 60 kg of nitrogen while the Mixture B contains 60 kg of phosphate and 40 kg of nitrogen. Formulate the problem as a linear programming problem to find out how many bags of each type a farmer should buy to get the desired amount of fertilizers at the minimum cost. (**Do not Solve**) [6 Marks]

(e) Write the dual problem associated with the following linear programming problem [5 Marks]

$$\begin{aligned} \text{Minimize} \quad & C = 4x_1 + 6x_2, \\ \text{Subject to} \quad & 20x_1 + 10x_2 \geq 1200, \\ & 10x_1 + 18x_2 \geq 2000, \\ & 5x_1 + 15x_2 \geq 3000, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

(f) Consider the following LP model for determining the levels of production of three products, A, B and C:

$$\begin{aligned} \text{Maximize} \quad & P = 6x_1 + x_2 + 2x_3 \\ \text{Subject to} \quad & x_1 + 2x_2 + 5x_3 \leq 16 \quad \text{(Resource 1)} \\ & 5x_1 + 2x_2 + x_3 \leq 20 \quad \text{(Resource 2)} \\ & -x_1 \quad\quad + 2x_3 \leq 2 \quad \text{(Resource 3)} \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

where x_1, x_2 and x_3 are the units of products A, B and C, respectively, to be produced, and x_4, x_5 and x_6 are slack variables. The solution of the LP by the Simplex method proceeds as follows:

	Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
STEP 1	x_4	1	2	5	1	0	0	16
	x_5	5	2	1	0	1	0	20
	x_6	-1	0	2	0	0	1	2
	P	-6	-1	-2	0	0	0	0

	Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
STEP 2	x_4	0	$\frac{8}{5}$	$\frac{24}{5}$	1	$-\frac{1}{5}$	0	12
	x_1	1	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	0	4
	x_6	0	2	7	1	0	1	18
	P	0	$\frac{7}{5}$	$-\frac{4}{5}$	0	$\frac{6}{5}$	0	24

Do the remaining steps(s) of the Simplex method to obtain the optimal solution. [5 Marks]

(g) Find the amount of an ordinary annuity consisting of 12 monthly payments of E100 that earn interest at 12% per year compounded monthly. [4 Marks]

(h) Find the present value of an ordinary annuity consisting of 24 monthly payments of E100 each and earning interest at 9% per year compounded monthly. [4 Marks]

(i) Use the method of diagonals to find the determinant of the following matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

[5 Marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) A certain country's production in the early years following World War II is described by the function

$$f(x,y) = 30x^{\frac{2}{3}}y^{\frac{1}{3}}$$

units, when x units of labour and y units of capital were used.

i. Compute f_x and f_y. [4 Marks]

ii. What is the marginal productivity of labour and the marginal productivity of capital when the amounts expended on labour and capital are 125 units and 27 units, respectively? [4 Marks]

iii. Should the government have encouraged capital investment rather than increasing expenditure on labour to increase the country's productivity? [2 Marks]

(b) Consider an agent who consumes Good A and Good B. We denote the amount of those goods by x and y, respectively. The utility function of this agent is given by

$$u(x,y) = x^{\frac{1}{2}}y^{\frac{1}{2}}.$$

Suppose that this agent has an income of 12 and does not save. A unit of Good A costs $E2$ and a unit of Good B costs $E1$. Use the method of Lagrange multipliers to maximize the utility of the agent. [10 Marks]

QUESTION B3 [*20 Marks*]

B3 (a) Robertson Controls manufactures two basic models of setback thermostats: a standard mechanical thermostat and a deluxe electronic thermostat. Robertson's monthly revenue (in hundreds of dollars) is

$$R(x,y) = -\frac{1}{8}x^2 - \frac{1}{2}y^2 - \frac{1}{4}xy + 20x + 60y$$

where x (in units of a hundred) denotes the number of mechanical thermostats manufactured and y (in units of a hundred) denotes the number of electronic thermostats manufactured each month. The total monthly cost incurred in producing these thermostats is

$$C(x,y) = 7x + 20y + 280$$

hundred dollars.

i. Find how many thermostats of each model Robertson should manufacture each month in order to maximize its profits. [9 Marks]

ii. What is the maximum profit? [1 Marks]

(b) Consider the function

$$f(x,y) = 4y^3 + x^2 - 12y^2 - 36y + 2$$

i. Find the critical point(s) of f. [5 Marks]

ii. Use the second derivative test to classify the nature of the critical point(s). [4 Marks]

iii. Find the relative extrema of f, if it exists [1 Mark]

QUESTION B4 [*20 Marks*]

B4 (a) Consider the matrix

$$A = \begin{bmatrix} 6 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

After applying several elementary row operation steps of the Gauss-Jordan method we obtain

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 0 & 0 & 1 \\ 0 & 1 & 6 & 0 & -1 & 2 \\ 0 & -4 & -23 & 1 & 0 & -6 \end{array}\right]$$

Complete the remaining steps of the Gauss-Jordan method to find the inverse of A. [8 Marks]

(b) Use the adjoint method to find the inverse of the matrix A given in (a) above. [12 Marks]

QUESTION B5 [*20 Marks*]

B5 A nutritionist advises an individual who is suffering from iron and vitamin-B deficiency to take at least 2400 milligrams (mg) of iron, 2100 mg of vitamin B1 (thiamine), and 1500 mg of vitamin B2 (riboflavin) over a period of time. Two vitamin pills are suitable, brand A and brand B. Each brand-A pill costs 6 cents and contains 40 mg of iron, 10 mg of vitamin B1, and 5 mg of vitamin B2. Each brand-B pill costs 8 cents and contains 10 mg of iron and 15 mg each of vitamins B1 and B2.

(a) Formulate the problem as a linear programming problem (LPP) that seeks to find the combination of pills that the individual must purchase in order to meet the minimum iron and vitamin requirements at the lowest cost. [6 Marks]

(b) Solve the linear programming problem using the *graphical method.* [14 Marks]

QUESTION B6 [*20 Marks*]

B6 (a) Find the best transportation schedule for the data given in the table below using the North-West corner rule and the Stepping Stone method.

From\To	1	2	3	Supply
A	2	12	8	450
B	3	6	11	250
Demand	160	210	330	700

[12 Marks]

(b) A department has four employees with five jobs to be performed. The time (in hours) each employee will take to perform each job is given in the effectiveness matrix. How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

	Jobs			
Employee	Job 1	Job 2	Job 3	Job 4
E1	23	45	41	31
E2	12	56	59	28
E3	33	31	53	25
E4	31	17	30	37

[8 Marks]

END OF EXAMINATION PAPER