UNIVERSITY OF ESWATINI

JANUARY 2019 RE-SIT EXAMINATION

IDE B.Com II,III, B.Ed (Sec.) II,III

Title of Paper : Quantitative Techniques

Course Number : MS202/MAT202

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.

2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.

3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.

4. Show all your working.

5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.

6. You can answer questions in any order.

7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) The monthly profit (in dollars) of Bond and Barker Department Store depends on the level of inventory x (in thousands of dollars) and the floor space y (in thousands of square feet) available for display of the merchandise, as given by the equation

$$P(x,y) = -0.02x^2 - 15y^2 + xy + 39x + 25y - 20,000$$

Compute $\dfrac{\partial P}{\partial x}$ and $\dfrac{\partial P}{\partial y}$ when $x = 4000$ and $y = 150$. Interpret your results. [5 Marks]

(b) Compute the partial derivatives f_x, f_y and f_{xx} corresponding to the function

$$f(x,y) = \frac{y}{x^2 + y^2}$$

[6 Marks]

(c) Find the relative extrema of the function

$$f(x,y) = 3x^2 - 4xy + 4y^2 - 4x + 8y + 4$$

[8 Marks]

(d) Consider the following transportation problem

From\To	1	2	3	4	Supply
A	9	4	3	12	270
B	11	13	10	4	90
C	8	10	1	7	160
Demand	10	20	230	260	520

     i. Find the initial allocation and cost using the North-West Corner Rule [3 Marks]

     ii. Show that the initial allocation is optimal. [4 Marks]

(e) A toy assembly company assemblies three types of toys - trains, trucks and cars, using three different operations. The daily limits on the available times for the three operations are 430, 460, and 420 minutes, respectively. The revenues per unit toy train, truck, and car are R3, R2, and R5, respectively. The assembly times per train at the three operations are 1,3, and 1 minutes, respectively. The assembly times per toy truck at the three operations are 2,0, and 4 minutes, respectively (a zero time indicates that the operation is not used). The corresponding times per toy car are 1,2, and 0 minutes, respectively.

Let x_1, x_2, and x_3 represent the daily number of units assembled of trains, trucks, and cars, respectively.

Formulate (*DO NOT SOLVE!*) the associated linear programming problem. [6 Marks]

(f) Write the dual problem associated with the following linear programming problem [5 Marks]

$$\text{Minimize} \quad C = 2x_1 + 5x_2$$
$$\text{Subject to} \ 2x_1 + x_2 \geq 12$$
$$x_1 + x_2 \geq 20$$
$$5x_1 + x_2 \geq 30$$
$$x_1 \geq 0, \ x_2 \geq 0$$

(g) Woodmasters manufactures chairs that can be bought fully assembled or in do-it-yourself (DIY) assembly kits. The demand equations that relate the unit prices, p and q, to the quantities demanded weekly, x and y, of the assembled and DIY kit versions of the chairs are given by

$$p = 200 - \frac{1}{4}x - \frac{1}{8}y \ \text{ and } \ q = 240 - \frac{1}{8}x - \frac{5}{8}y$$

Find the weekly total revenue function $R(x, y)$ [3 Marks]

> ## SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) Siza is a football player who believes that his playing career will last 10 years. He deposits E30,000 at the end of every year for 10 years in an account paying 6% compounded annually. How much will he have after 10 years?   [4 Marks]

(b) A company will need to replace a piece of equipment at a cost of R900,000 in 10 years. To have this money available in 10 years' time, a sinking fund is established by making equal monthly deposits into an account paying 4% compounded monthly.   [6 Marks]

(c) A family bought a house for E64,000 cash 10 years ago and took out a 30-year mortgage at 4.5% on the unpaid balance. What is the current unpaid balance on the mortgage? [10 Marks]

QUESTION B3 [*20 Marks*]

B3 The Mafutseni Ranch is considering buying two different brands of chicken feed and blending them to provide a good, low-cost diet for its chickens. Each brand of feed contains, in varying proportions, some or all of the three nutritional ingredients essential for fattening chickens.

Each kilogram of brand 1 contains 5 grams of ingredient A, 4 grams of ingredient B and 0.5 grams of ingredient C. Each kilogram of brand 2 contains 10 grams of ingredient A, 3 grams of ingredient B, but nothing of ingredient C. The brand 1 feed costs the Ranch 20c a kilogram, while the brand 2 feed costs 30c a kilogram. The minimum monthly requirement per chicken is: 90 grams of ingredient A; 48 grams of ingredient B and 1.5 grams of ingredient C.

(a) Formulate an LP model to help the rancher decide how to mix the two brands of chicken feed so that the minimum monthly intake requirement for each nutritional ingredient is met at minimum cost.   [6 Marks]

(b) Use the graphical approach to solve this model   [14 Marks]

QUESTION B4 [*20 Marks*]

B4 (a) A certain country's production in the early years following World War II is described
by the function
$$f(x,y) = 60x^{\frac{1}{3}}y^{\frac{2}{3}}$$
units, when x units of labour and y units of capital were used.

   i. Compute f_x and f_y.   [4 Marks]
   ii. What is the marginal productivity of labour and the marginal
   productivity of capital when the amounts expended on labour and
   capital are 125 units and 8 units, respectively?   [4 Marks]
   iii. Should the government have encouraged capital investment
   rather than increasing expenditure on labour to increase the
   country's productivity?   [2 Marks]

   (b) Sipho consumes only two fruits, apples and bananas. Denote his consumption of these
   fruits by x and y, respectively. Sipho's utility level is given by the following utility
   function:
   $$u(x,y) = x^{\frac{1}{3}}y^{\frac{2}{3}}$$

   In the market, apples and bananas are both sold at $E1$ per unit. Sipho's income is E300
   and he does not save. Solve Sipho's utility maximisation problem using the method of
   Lagrange multipliers method.   [10 Marks]

QUESTION B5 [*20 Marks*]

B5 (a) Solve the following linear system using the Gauss-Jordan method,

$$x + 3y + 3z = 16,$$
$$x + 4y + 3z = 18,$$
$$2x + 7y + 7z = 37.$$

                                                                            [10 Marks]

   (b) Consider a case where there are five jobs (J1, J2, J3, J4 and J5) that must be assigned
   to five workers (W1, W2, W3, W4 and W5), one job per worker. The matrix below
   shows the cost of assigning a certain worker to a certain job. Use the Hungarian method
   to find the assignment that minimizes the total cost of the assignment.   [10 Marks]

	J1	J2	J3	J4	J5
W1	83	21	16	73	24
W2	97	43	8	91	22
W3	86	95	22	42	13
W4	83	59	75	44	10
W5	70	24	88	48	64

QUESTION B6 [*20 Marks*]

B6 Use the Simplex method to solve the following linear programming problem

$$\text{Maximize} \quad 3x_1 + 4x_2 + 3x_3$$
$$\text{Subject to} \quad x_1 + 4x_2 + x_3 \le 16$$
$$3x_1 + 4x_2 + x_3 \le 40$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

[20 Marks]

END OF EXAMINATION PAPER