UNIVERSITY OF ESWATINI

RE-SIT EXAMINATION, 2019/2020

BASS

Title of Paper : Elementary Quantitative Techniques I

Course Number : MAT101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
   (a) Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
   (b) Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
2. Show all your working.
3. Start each new major question (A1, A2, B2, ..., B7) on a new page and clearly indicate the question number at the top of the page.
4. Non-programmable calculators may be used (unless otherwise stated).

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A
ANSWER ALL QUESTIONS

6

QUESTION A1

(a) Use synthetic division to find the quotient and remainder when
$P(x) = x^4 - x^3 - 3x^2 + 4x + 4$ is divided by $d(x) = x + 2$ (5 marks)

(b) Solve the simultaneous equations
$$5x - 2y = 9$$
$$4x + 5y = -6$$
(5 marks)

(c) Find the sum of first 10 terms of the following progressions
(i) $3, 7, 11, 15, \ldots$
$$\frac{-1}{3}, \frac{2}{3}, \frac{-4}{3}, \frac{8}{3}, \ldots$$
(5 marks)

(d) Find an equation of a straight line through the point $(-1, -1)$ and perpendicular to
$2x - 3y = 12$ (5 marks)

QUESTION A2

(a) Consider the matrices
$$A = \begin{pmatrix} 4 & -3 \\ 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -5 \\ 3 & 6 \\ 1 & -4 \end{pmatrix} \qquad C = \begin{pmatrix} 8 & -2 & 3 \\ -1 & 5 & -4 \end{pmatrix}$$
Find (i) $|A|$ (ii) $A^T B^T$ (iii) BC (iv) $C^T B^T$
(12 marks)

(b) Use Cramer's rule to solve
$$3x + 4y + z = 7$$
$$4x + 3y - 2z = -5$$
$$x + 2y + 4z = 20$$
(8 marks)

SECTION B

QUESTION B3

(a) (i) Find the first five terms of the infinite binomial expansion

$$(1+2x)^{20}$$ (5 marks)

(ii) Find the middle term of the binomial expansion

$$\left(2x^2+\frac{1}{x}\right)^{12}$$ (5 marks)

(b) Find the equations of the following lines and express your answer in the form $y = mx + c$

(i) Line through $(-1, 4)$ which is parallel to the line through $(1, -1)$ and $2, 3$ (5 marks)

(ii) Line through $(-1, -2)$ which is perpendicular to the line

$$2x + 3y - 4 = 0$$ (5 marks)

QUESTION B4

(a) Given the following matrices

$$A=\begin{pmatrix}-3 & 5\\ 2 & -1\end{pmatrix} \qquad B=\begin{pmatrix}-5 & 1 & 2\end{pmatrix}$$

$$C=\begin{pmatrix}1 & 3\\ -1 & 0\\ 4 & -2\end{pmatrix} \qquad D=\begin{pmatrix}-2 & 9 & 6\\ -3 & 3 & 4\\ 2 & -2 & 1\end{pmatrix} \qquad E=\begin{pmatrix}3 & -2 & 4\\ 2 & 1 & 5\end{pmatrix}$$

Compute

(i) AE
(ii) BC
(iii) $E^T + C$
(iv) BE^T
(v) $|D|$

(10 marks)

(b) (i) Given that $\sin A = \frac{3}{5}$ and $\cos A$ is negative

Find $\sin 2A$ and $\tan 2A$ (5 marks)

(ii) Prove the following identity

$$\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$$ (5 marks)

QUESTION B5

(a) Prove the following identities

(i) $\sec x - \sin x \tan x = \cos x$

(ii) $\sin\theta + \cos\theta\cot\theta = \cos ec\theta$

(10 marks)

(b) Given that $\cot\theta = -2$ and that the terminal side is in quadrant IV, find the remaining five trigonometric ratios of θ. (10 marks)

QUESTION B6

(a) Use synthetic division to find the quotient and the remainder

(i) $\left(x^3 - 7x^2 - 13x + 3\right) \div \left(x + 2\right)$

(ii) $\left(2x^4 + 3x^2 - 1\right) \div \left(x - 3\right)$ (14 marks)

(b) (i) Find the 10^{th} constant term of the binomial expansion

$$\left(x + \frac{1}{x^2}\right)^{12}$$

(ii) Use the quadratic formula to solve

$$6x^2 - 13x - 5 = 0$$ (6 marks)

QUESTION B7

(a) (i) Use synthetic division to find the quotient and remainder

$$\left(4x^4 - x^2 - 6x - 9\right) \div \left(x - 3\right)$$ (5 marks)

(ii) Use long division to find the quotient and remainder

$$\left(x^3 - 7x^2 - 13x + 3\right) \div \left(x + 2\right)$$ (5 marks)

(b) Prove the following identities

(i) $\left(\tan\theta + \cot\theta\right)\left(\cos\theta + \sin\theta\right) = \csc\theta + \sec\theta$ (5 marks)

(ii) $\sec x - \sin x \tan x = \cos x$ (5 marks)