University of Swaziland

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Re-sit Examination – January/February 2020

BSc I, BEng I, BEd I, BASS I, BSc IT I, BSc IS I, BSc Comp. Sci. Ed. I

Title of Paper	:	Algebra, Trigonometry & Analytic Geometry
Course Number	:	MAT111
Time Allowed	:	Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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Section A

Answer ALL Questions in this section

A.1 a. Solve for x given

$$e^{2x-5} = 9\,999$$

leaving your answer correct to 2 d.p. [3 marks]

b. Evaluate

$$2 + 3i - (1 - 2i)^2$$

and leave your answer in the form $a + ib$. [4 marks]

c. Find the angle between the vectors $\boldsymbol{A} = -4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\boldsymbol{B} = 8\hat{j} - 2\hat{k}$, leaving your answer in degrees correct to 2 d.p. [5 marks]

d. Find the value of

$$\sum_{n=0}^{90} 50(1.04)^n$$ [4 marks]

correct to 2 d.p.

e. Consider the equation of a circle

$$x^2 + y^2 - 12x = 0.$$

i. Find the coordinates of the centre and radius [3 marks]

ii. Make a sketch of the circle. [2 marks]

f. Evaluate the determinant

$$\begin{vmatrix} -2 & 0 & 3 \\ 1 & -4 & 0 \\ 2 & 5 & -1 \end{vmatrix}.$$ [5 marks]

g. Find the quotient and remainder of

$$\frac{x^4 - 2x^3 + x^2 - 5x + 10}{x^2 - 2}.$$ [5 marks]

h. In the binomial expansion of

$$\left(x + \frac{2}{x^2}\right)^{22},$$

find the first 3 terms and simplify term by term. [5 marks]

i. Find the *exact* value of

$$\left(\cos 795^0 - \sin 795^0\right)^2.$$ [4 marks]

2.1

Section B

Answer ANY 3 Questions in this section

B.2 a. Consider the complex number

$$\Omega = \sqrt{3} - i.$$

Find

i. the modulus of Ω [2 marks]

ii. the argument of Ω in radians [3 marks]

iii. the polar form of Ω [1 marks]

iv. Ω^{12} using *de Moivre's theorem* and express in the form $a + ib$. [4 marks]

b. Find the remainder when the polynomial

$$P(x) = 2x^4 - 3x^3 - 12x^2 + 7x + 6$$

is divided by

i. $x + 2$ [2 marks]

ii. $x - 4$ [2 marks]

c. Hence, or otherwise, factorise

$$P(x) = 2x^4 - 3x^3 - 12x^2 + 7x + 6.$$ [6 marks]

B.3 a. Simplify

$$\left(\cos \tfrac{5}{12}\pi + i \sin \tfrac{5}{12}\pi\right)\left(\cos \tfrac{11}{12}\pi - i \sin \tfrac{11}{12}\pi\right).$$ [5 marks]

b. Find the *general solution* of the equation

$$2\sin^2\theta = 1 + \cos\theta,$$

expressing your answer in radians. [5 marks]

c. Prove each of the following trigonometric identities:

i. $1 - \dfrac{\sin^2 A}{1 + \cos A} = \cos A$ [4 marks]

ii. $\dfrac{\sin 3\theta - \sin\theta}{\cos^2\theta - \sin^2\theta} = 2\sin\theta$ [6 marks]

B.4 a. In the binomial expansion of

22

$$\left(x^3 - \frac{y}{x^2}\right)^{20}$$

find the

i. middle term [4 marks]

ii. term involving $\frac{1}{x^{15}}$ [5 marks]

b. Find the first 4 terms of the binomial expansion of

$$\sqrt{1 - 2x^2}.$$ [6 marks]

c. The second term of a *geometric progression (GP)* is −5. If the sum of an infinite number of terms of the GP is $\frac{20}{3}$, find the first term and the common ratio. [5 marks]

B.5 a. Simplify

$$\log_b b^4 + \ln e^{2m-1} + 4\log\sqrt{(0.00001)^m}.$$ [3 marks]

b. Solve for x given

i. $2^x + 2^{-x} = \frac{5}{2}$ [5 marks]

ii. $\log_2(x+2) + \log_2(x-5) = 3$ [5 marks]

c. A principal sum of E15 000 is invested in an account that pays 9.9% p.a. compounded daily. After a period of t years, the value of the account is given by

$$A(t) = 15\,000\left(1 + \frac{0.099}{365}\right)^{365t}.$$

i. Find the value of the account after 4.5 years. [2 marks]

ii. Find the time required for the principal amount to grow to E30 000. [5 marks]

B.6 a. A parabola, whose axis is paralell to the x-axis, passes through $(-2, 4)$, $(-3, 2)$ and $(-11, -2)$.

i. Find the equation of the parabola [5 marks]

ii. Hence, find the coordinates of the vertex and focus. [5 marks]

b. Use mathematical induction to prove that

$$P(n) = 2^{n+2} + 3^{2n+1}$$

is always divisible by 7, where the integer $n \geqslant 0$ [10 marks]

END OF EXAMINATION