University of Eswatini

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Resit Examination, January 2020

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : MAT211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. SECTION A(COMPULSORY): 40 MARKS
      Answer ALL QUESTIONS.
   b. SECTION B: 60 MARKS
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Begin each major question (A1, B2, etc) on a new page.
3. Each question in Section B is worth 20%.
4. Show all your working.
5. Non programmable calculators may be used (unless otherwise stated).
6. Special requirements: None.
7. Indicate your program next to your student ID number.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A: Answer All Questions

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A1.

(a) i. Define a point of inflection of a function $f(x)$. [2]

ii. State the Second Derivative Test for concavity. [3]

iii. Determine the open intervals on which the graph of $f(x) = -x^3+6x^2-9x-1$ is concave upward or concave downward. [6]

iv. Use appropiate rules to find the limits of the following functions.

A. $\lim_{x\to\infty} \frac{2x-1}{3x+2}$. [2]

B. $\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$. [6]

(b) i. The region bounded by the curves $y = 2x$, $y = 3$ and the y-axis is rotated about the y-axis to generate a solid of revolution. Set up and evaluate, the integral for the volume of the solid. [5]

ii. Find the area of the region enclosed by the curves, $x = y^3$ and $x = y^2$. [4]

iii. Set up but do not evaluate, an integral for the area of the surface obtained by rotating the curve $y = \ln x$, over the interval $1 \le x \le 3$, about the x-axis.[3]

(c) i. Compute the first 3 terms in the sequence of partial sums for the series $\sum_{n=1}^{\infty} n2^n$. [3]

ii. Define a Taylor series generated by a function $f(x)$ at a point $x = c$. [3]

iii. Use the Alternating Series Test to show that the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+3}}{n^3+4n+1}$ converges . [3]

Section B: Answer Three(3) Questions Only

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B2.

Consider the function $f(x) = \dfrac{x^2+4}{2x}$.

(a) Identify the domain of $f(x)$. [1]

(b) Find and **classify** all critical points of $f(x)$. [4]

(c) Find intervals where $f(x)$ is increasing and where it is decreasing. [4]

(d) Find possible points of inflection, if any occur and determine concavity of the graph. [3]

(e) Identify any asymptotes that may exist. [3]

(f) Sketch the graph of $f(x)$ labelling all major points found above including intercepts if any occur. [5]

B3.

(a) Sketch and find the area of the region bounded by the graphs of $y = x^2+1$, $y = x$, $x = -1$, and $x = 2$. [10]

(b) Find volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about,

i. the x-axis, [5]

ii. the y-axis. [5]

B4.

(a) Find the arc length of the graph of $y = \frac{2}{3}(x^2+1)^{3/2}$ over the interval $[1,\ 4]$. [10]

(b) Find the area of the surface obtained by rotating the curve $x = \dfrac{y^4}{4} + \dfrac{1}{8y^2}$ over the interval $1 \le y \le 2$ about the y-axis. [10]

B5. 30

(a) i. Perform an index shift so that the series $\sum_{n=7}^{\infty} \frac{4-n}{n^2+1}$ starts at $n = 3$. [3]

ii. Show that geometric series $\sum_{n=0}^{\infty} 3^{2+n} 2^{1-3n}$, converges to $\frac{144}{5}$. [7]

(b) Find the interval and radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{4^{1+2n}}{5^{n+1}} (x+3)^n$. [10]

B6.

(a) Find the Taylor series for the function $f(x) = e^{-6x}$ about $x = -4$. [10]

(b) i. Define the n^{th} Taylor Polynomial for a function $f(x)$ about a point $x = c$. [3]

ii. Find the Taylor polynomials P_0, P_1, and P_2 for $f(x) = \ln x$ about $c = 1$. [7]

END OF EXAMINATION