University of Eswatini

Main Examination, 2019/2020

B.Ed (Pri.), (Sec.) II; B.Sc II

Title of Paper : Mathematics for Scientists

Course Number : MAT215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.

6. You can answer questions in any order.

7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Which of the following matrices is/are in reduced row echelon form?

$$A = \begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[3 marks]

(b) Find the area of the triangle ABC with vertices $A(7, -8)$, $B(-2, -6)$ and $C(1, 5)$. [4 marks]

(c) Show that the vectors $p = (3, -2, 0)$, $q = (2, -4, -2)$ and $r = (1, 2, 2)$ are coplanar. [4 marks]

(d) State Mean Valued Theorem. [3 marks]

(e) If $f(x) = x^3 - 7x + 6$. Find the number c that satisfies the condition of the Mean Valued Theorem for $f(x)$ on $[1, 3]$. [4 marks]

(f) If $f(x, y) = xy \cos x$. Find $f_x(\frac{\pi}{2}, 1)$. [3 marks]

(g) Find the critical point(s) of $f(x, y) = x^3 + y^2 - 3x - y$. [4 marks]

(h) Evaluate $\int_1^2 \int_0^x (2xy + 3) dy dx$. [5 marks]

(i) Let $y(t)$ be the unknown. Identify the order, degree and linearity of the following equations.

i. $3y' + (t + 4)y = t^2 + y''$, where $y' = \frac{dy}{dt}$. [1,1,1 marks]

ii. $(y^{(4)})^2 + \sqrt{t}(y''')^4 + \cos t = e^y$, where $y^{(4)} = \frac{d^4y}{dt^4}$ and $y''' = \frac{d^3y}{dt^3}$. [1,1,1 marks]

(j) If $f(x, y) = x^2y^2 + xy^3$. Show that the functions $f(x, y)$ is homogeneous and find the degree. [4 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) i. Find the reduced row echelon form of the matrix below:

$$\begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}.$$

[5 marks]

ii. The row echelon form of the augmented matrix of a linear system is given by;

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right).$$

The linear system is in the variable x, y and z.
Find the solution for x, y and z. [5 marks]

(b) If the cosine of the angle between $u = i + 2j + 2k$ and $v = i - 4j + mk$ is $\frac{1}{3}$, find the value of the parameter m. [10 marks]

QUESTION B3 [*20 Marks*]

B3 (a) State Rolle's Theorem. [3 marks]

(b) Let $g(x) = x^3 + 2x^2 - x - 1$. Find a number c in $(-2, 1)$ such that the tangent to the graph of $y = g(x)$ is horizontal at $x = c$. [6 marks]

(c) Find the first four terms of the Taylor series expansion of $\sin x$ about $x = 0$. [6 marks]
Use this series to evaluate $\lim_{x \to 0} \frac{\sin x - x}{x^3}$. [5 marks]

QUESTION B4 [*20 Marks*]

B4 (a) A sporting goods manufacturer produces regulation soccer balls at two plants.
The costs of producing x_1 units at Location 1 and x_2 units at Location 2
are given by

$$C_1(x_1) = 0.02x_1^2 + 4x_1 + 500$$

and

$$C_2(x_2) = 0.05x_2^2 + 4x_2 + 275$$

respectively. If the product sells for $E50$ per unit, then the profit function for the product is given by

$$P(x_1, x_2) = 50(x_1 + x_2) - C_1(x_1) - C_2(x_2).$$

Evaluate $P(250, 150)$. [8 marks]

(b) If $f(x, y, z) = 3x^2y - 5xyz - 10yz^2$, find $f_{xy}(1, 1, 1)$. [4 marks]

(c) If $z = \sin u + \cos v$, where $u = x^2y$ and $v = 2x + 3y$. By using chain rule, find $\frac{\partial^2 z}{\partial y^2}$. [8 marks]

QUESTION B5 [*20 Marks*]

B5 A company makes two substitute products whose demand functions are given by

$$x_1 = 200(p_2 - p_1) \qquad \text{and} \qquad x_2 = 500 + 100p_1 - 180p_2$$

where p_1 and p_2 are the prices per unit (in Emalangeni) and x_1 and x_2 are numbers of units sold. The costs of producing the two products are $E0.50$ and $E0.75$ per unit respectively. Find the prices that will yield a maximum profit. [20 marks]

QUESTION B6 [*20 Marks*]

B6 (a) Find the solution of $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$, $y(0) = -1$. [8 marks]

(b) Consider the differential equation $y'' - 5y' + 6y = 0$, where $y' = \frac{dy}{dx}$.

i. Find the general solution. [6 marks]

ii. If $y(0) = -1, y'(0) = 5$, find the particular solution. [6 marks]

END OF EXAMINATION