UNIVERSITY OF ESWATINI

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RESIT EXAMINATION, 2019/2020

B.A.S.S. II, B.Ed (Sec.) II, B.Sc. II, B.Eng. II

Title of Paper : Linear Algebra

Course Number : MAT221

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) i) Determine whether the vectors $\bar{v}_1 = (0, 1, -2)$, $\bar{v}_2 = (-3, 0, 1)$, and $\bar{v}_3 = (1, 2, -1)$ are linearly independent or linearly dependent. [4]

ii) Let V_{nn} be the vector space of $n \times n$ matrices. Determine whether the transformation $T(A) = A^T - 3A$ is linear transformation or not. [4]

b) i) Find $|A|$ and $|3A^T|$, given that $A = \begin{bmatrix} 1 & 6 & 1 & -4 \\ 0 & -4 & 0 & 7 \\ 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 5 \end{bmatrix}$. [3]

ii) Hence determine if the matrix A is invertible or not. [2]

c) i) Determine the characteristic polynomial of the matrix $A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & 1 & 0 \\ -4 & 0 & -3 \end{bmatrix}$. [2]

ii) Hence find the corresponding eigenvalues. [3]

d) i) Express $C = \begin{bmatrix} -1 & 1 \\ 0 & 3 \end{bmatrix}$ as a product of elementary matrices. [6]

ii) Find $2A^{-7}$, where $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. [4]

e) Solve the following system, using Gauss-Jordan elimination. [6]

$$2x_1 + x_2 = 18$$
$$3x_1 + 6x_2 = 9$$

f) Verify the Cayley-Hamilton theorem for the matrix [6]

$$A = \begin{bmatrix} 1 & 4 \\ 0 & -3 \end{bmatrix}$$

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Determine whether the vectors $\mathbf{v}_1 = (8, 1, -3)$, $\mathbf{v}_2 = (4, 0, 1)$ are linearly dependent or linearly independent in $\mathbb{R}^3$. [10]

b) Do the vectors $\mathbf{v}_1 = (1, 2, 1)$, $\mathbf{v}_2 = (2, 9, 0)$, $\mathbf{v}_3 = (3, 3, 4)$; form a basis for $\mathbb{R}^3$? [10]

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QUESTION B3 [*20 Marks*]

a) Solve the system of equations [12]

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= -2 \\ 4x_1 - 5x_2 + 8x_3 &= 0 \\ -3x_1 + 3x_2 - 3x_3 &= 1. \end{aligned}$$

b) Prove that a square matrix A is invertible if and only if $\kappa = 0$ is not an eigenvalue of A. [8]

QUESTION B4 [*20 Marks*]

a) Suppose that the matrices A and B are both symmetric with the same size, show that $A+B$ is symmetric. [7]

b) Find a matrix P that diagonalizes the matrix $A = \begin{bmatrix} -2 & 0 & 0 \\ 9 & 1 & 0 \\ 8 & 6 & -3 \end{bmatrix}$ and hence write down an expression in terms of the matrix P that can be used to evaluate A^6. [13]

QUESTION B5 [*20 Marks*]

a) Find bases for the eigenspaces of the matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. [10]

b) Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Verify Cayley-Hamilton theorem. [10]

QUESTION B6 [*20 Marks*]

a) Define $P : \mathbb{C}^3 \to \mathbb{C}^2$ by describing the output of the function for a generic input with the formula

$$P\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ x_1 - 5x_2 \end{bmatrix}$$

Determine whether the transformation is linear or not. [12]

b) Prove that If $T : V \to W$ is a linear transformation, then:

i) $T(\mathbf{0}) = \mathbf{0}$ [4]

ii) $T(-\mathbf{u}) = -T(\mathbf{u})$ [4]

END OF EXAMINATION PAPER