UNIVERSITY OF ESWATINI

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RE-SIT EXAMINATION, 2019/2020

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Foundations of Mathematics

Course Number : MAT231

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B2, ..., B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A (Answer ALL Questions in this Section)

Question A1 [20 Marks]

(a) Let $A = \{1,2,3\}$ and $B = \{x,y,z\}$. Let R be the relation from A into B defined by

$$R = \{(1,y),(1,z),(3,y)$$

(i) Is R a function from A into B? Why or why not? (2)

(ii) Write down the domain and range of R. (2)

(b) (i) Give a clear definition of a *partial order relation* on a set A. (3)

(ii) Let X be a set and let $\mathscr{P}(X)$ be the power set of X. Show that the subset relation is a partial order relation on $\mathscr{P}(X)$. (5)

(c) (i) Define a *bijection* $f : A \to B$. (2)

(ii) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $(x) = 2x - 3$ is an bijection. (6)

Question A2 [20 Marks]

(a) (i) Give clear definitions of a *tautology* and a *contradiction*. (2)

(ii) Use truth tables to show that $p \wedge \neg p$ is a contradiction and that $p \vee \neg p$ is a tautology. (4)

(b) Construct a truth table for the proposition $(\neg p \vee q) \to p$. (4)

(c) Use a truth table to prove that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$. (4)

(d) Negate each of the following statements. (6)

(i) $\exists x \forall y, p(x,y)$ (ii) $\forall x \exists y, p(x,y)$ (iii) $\exists x \exists y \forall z, p(x,y,z)$

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Section B (Answer any three (3) Questions in this Section)

Question B3 [20 Marks]

(a) Let $U = \{n \in \mathbb{N} : 1 \leq n \leq 10\}$ and let $E = \{m \in U : m \text{ is even}\}$, $O = \{m \in U : m \text{ is odd}\}$, and $P = \{p \in U : p \text{ is prime}\}$. Find the following sets.

(i) $E \cup O$ (ii) $E \cap P$ (iii) $P \setminus O$ (iv) E^c (8)

(b) Let $A = \{a, b\}$. Write down $\mathscr{P}(A)$, the *power set* of the set A. (4)

(c) Let A be any set and let $\emptyset$ be the emptyset. Show that $\emptyset \subseteq A$. (4)

(d) Prove: *If $A \subseteq B$, then $A \cup B = B$.* (4)

Question B4 [20 Marks]

(a) Which of the following arguments are valid and which are not? For the valid arguments, state if *modus tollens* or *modus ponens* was used. For the invalid arguments, state if the *inverse error* or the *converse error* was made.

(i) If I have studied well, then I will pass the exam. I will pass the exam. Therefore, I have studied well. (2)

(ii) If I have studied well, then I will pass the exam. I will fail the exam. Therefore, I have not studied well. (2)

(iii) If I have studied well, then I will pass the exam. I have not studied well. Therefore, I will fail the exam. (2)

(iv) If I have studied well, then I will pass the exam. I have studied well. Therefore, I will pass the exam. (2)

(b) Prove without using truth tables that $(p \wedge q) \wedge \neg(p \vee q) \equiv c$ (4)

(c) Use truth tables to prove the following: $\neg(p \vee q) \equiv \neg p \wedge \neg q$. (4)

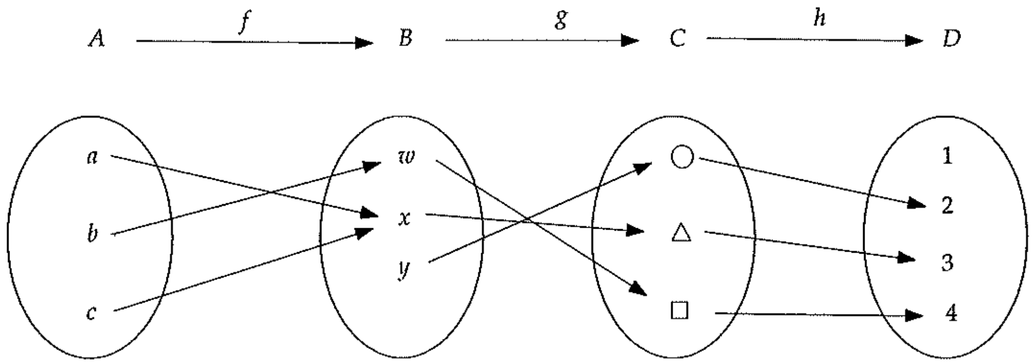
(d) Write down the contrapositive of the statement: *For every $x \in \mathbb{R}$, if $x(x+1) > 0$ then $x > 0$ or $x < -1$.* (4)

Question B5 [20 Marks]

(a) Find the domain of each function below. (4)

(i) $f(x) = \dfrac{1}{(x-2)(x-3)}$ (ii) $f(x) = \ln(x^2 - 1)$

(b) consider the functions f, g, and h defined in the picture below.



Determine which functions are (i) injective, (ii) surjective, and (iii) invertible. (6)

(c) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x - 3$ and $g(x) = \sqrt{x-1}$. Find $(g \circ f)(x)$. (4)

(d) Let $f : A \to B$ and $g : B \to C$ be one-to-one functions. Prove that $g \circ f : A \to C$ is also a one-to-one function. (6)

Question B6 [20 Marks]

(a) Prove: *For $a, b, c \in \mathbb{Z}$ with $a \neq 0$, if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.* (4)

(b) Prove: *There is no rational number x such that $x^2 = 2$.* (6)

(c) Prove: *For any integer n, $n^3 + n$ is even.* (6)

(d) True or False? (If true, give a proof. If false, explain why.): *For all real numbers $x > 0$, $x > \dfrac{1}{x}$.* (4)

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Question B7 [20 Marks]

(a) Use mathematical induction to prove:

For all integers $n \geq 0$, $3^{2n} + 7$ is divisible by 8.

(10)

(b) Use strong induction to prove:

For all integers $n \geq 2$, either n is prime or n can be written as a product of prime numbers.

(10)

END OF EXAMINATION