# University of Eswatini

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## Supplementary Examination, January 2020

## B.Sc III, B.A.S.S III, B.Ed III

**Title of Paper** : Real Analysis

**Course Code** : MAT331/M331

**Time Allowed** : Three (3) Hours

**Instructions**

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      **Submit solutions to ONLY THREE questions in Section B.**
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

# SECTION A: ANSWER ALL QUESTIONS

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## Question 1

(a) (i.) Give an example of a set which is a neighbourhood of each of its points with the exception of two points. [3]

(ii.) Explain what is means to say that the set $A$ is bounded above and define sup(A). [3]

(iii.) Define uniform continuity of a function. [3]

(b) (i) Prove that $x \leq |x|$ and $-x \leq |x|$. [3]

(ii.) Using the concept from subsequences show that $(-1)^n$ is a divergent sequence. [4]

(iii.) Prove that every finite set is bounded. [4]

(iv.) prove that an open interval is a neighbourhood of each of its points. [4]

(v.) Determine the lim $sup\ a_n$ and lim $inf\ a_n$ if $a_n = n\sin^2(\frac{\pi n}{2})$ [4]

(vi.) Prove that the sequence whose $n^{th}$ term is

$a_n = \sqrt{n+1} - \sqrt{n}$ is monotone. [4]

(c) Show that $f(x) = 3x^2 - 2x + 1$ is continuous at $x = 5$ [4]

(d) If $a_n = 2 + \dfrac{(-1)^n}{n^2}$, find the least positive integer $m$ such that

$$|a_n - 2| < \frac{1}{10^4} \quad \forall n > m.$$ [4]

# SECTION B: ANSWER ANY 3 QUESTIONS

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## Question 2

(a) Show that a non-empty finite set is not a neighbourhood of any point. [6]

(b) If $A$ and $B$ are bounded subsets of $\Re$ and the set $A + B = \{x + y : x \in A \text{ and } y \in B\}$, prove that $\sup(A + B) = \sup A + \sup B$. [7]

(c) Prove that supremum of a subset of a set if it exit is Uniqueness. [7]

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## Question 3

(a) Prove (by $\epsilon - n$ definition) that $\lim_{n\to\infty} \frac{4n^3 + 3n}{n^3 - 6} = 4$. [10]

(b) Prove that the sequence $n!$ is not convergent. [10]

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## Question 4

(a) If $a_n > 0 \ \forall n$ and $\lim_{n\to\infty} a_n \neq 0$, then prove that the series $\sum a_n$ diverges to $\infty$. [8]

(b) Discuss the convergence of $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \cdots$. [6]

(c) Test the convergence of the series $\sum \left(\frac{n}{n+1}\right)^{n^2}$ or $\sum \left(1 + \frac{1}{n}\right)^{-n^2}$. [6]

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## Question 5

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(a) Discuss the continuity of the function $f(x) = [x]$, (greatest integer function), at the point $x = 1$. If it is discontinuous at $x = 1$ what type of discontinuity is it? [10]

(b) Show that

$$f(x) = \begin{cases} x^2 - 1 & \text{when } x \geq 1 \\ 1 - x & \text{when } x < 1 \end{cases}$$

has no derivative at $x = 1$. [10]

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## Question 6

(a) Show that the function defined by $f(x) = x^2$ is uniformly continuous on $[-2, 2]$. [10]

(b) Verify $L(f, \Delta) \leq U(f, \Delta)$, (Lower and upper sum), using the following: $f(x) = x^2$, $[0, 1]$ and partition $\Delta = \{0, \frac{1}{2}, 1\}$. [10]

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End of Examination Paper