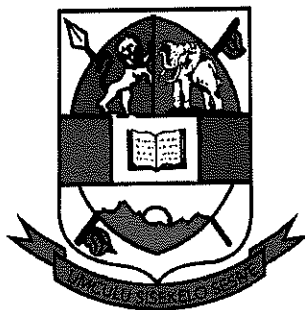


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UNIVERSITY OF SWAZILAND



MAIN EXAMINATION, 2019/2020

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**BASS IV, B.Ed (Sec.) IV, B.Sc IV**

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**Title of Paper** : COMPUTATIONAL METHODS

**Course Number** : MAT415

**Time Allowed** : Three (3) Hours

**Instructions**

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

**Special Requirements: NONE**

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

**SECTION A [40 Marks]: ANSWER ALL QUESTIONS****QUESTION A1 [40 Marks]**

- A1 (a) For each of the following statements (i -xv): Write your answer as TRUE if the statement is entirely true or FALSE if the statement is not entirely true.
- i. LaTeX is an open source code.
  - ii. `\quad` is a command in LaTeX for quadratic.
  - iii. The `%` sign is used for making comments in MATHEMATICA.
  - iv. The title page of a thesis must display references cited in the thesis.
  - v. The *for loop* repeats a statement or a group of statements with conditions based on any variable.
  - vi. The command **NDSolve** is used for differentiating functions in MATHEMATICA.
  - vii. Inaccurate citation is a form of plagiarism.
  - viii. A thesis can be defined as a proposition to be maintained or proved.
  - ix. There are predefined commands instructing LaTeX which part of the content makes the abstract.
  - x. In LaTeX, layout standards are contained in class files which have **.cls** as their names.
  - xi. An empty line starts a new paragraph in LaTeX.
  - xii. References are automatically generated in LaTeX.
  - xiii. `#` is a special character in LaTeX. One would need to use a backslash before the character if they intend to use it.
  - xiv. The command `\frac{dy}{dx}` generates  $\frac{\partial y}{\partial x}$  in LaTeX.
  - xv. The command `\int f(x) dx` generates  $\int_a^b f(x)dx$  in LaTeX.

[15 marks]

- (b) i. State two (2) components of an ideal thesis. [2 marks]  
ii. State any four (4) components of an **abstract of a thesis**. [4 marks]
- (c) Spectral radius of a matrix  $A$  is given by

$$\rho(A) := \max\{|\lambda| : \lambda \text{ is an eigenvalue of } A\}.$$

- i. State any two (2) LaTeX packages that would be required to typeset the definition above. [2 marks]
- ii. Write the above definition in LaTeX form, using an appropriate environment. [6 marks]
- (d) Using a *for loop* and a programming language of your choice, write a code to build a  $k$  by  $j$  matrix with entries

$$a_{kj} = \frac{1}{k+j-1} \quad k, j \in [0, 10].$$

[6 marks]

- (e) Write a pseudo code or code in MATHEMATICA for solving the initial value problem:

$$\begin{aligned}x' &= 0.2x(1-x) - 0.005xy, \\y' &= 5 - 2y, \\x(0) &= 100, \quad y(0) = 200, \quad t \in [0, 10].\end{aligned}$$

[5 marks]

**SECTION B: ANSWER ANY THREE QUESTIONS****QUESTION B2 [20 Marks]**

B2 Precision, clarity and compactness in scientific writing is very important. Moreover, there is no need for complexity. The paragraph below is an excerpt from a student's thesis:

*The existence of optimal control problem and the optimality system were established with an excellent optimal control problem formulated as follows below:*

$$\max_{x,V} J(x, V) = \psi(x(t_f)) + \int_0^{t_f} L(t, x(t), V(t)) dt$$

subject to the following conditions shown below

$$\begin{aligned} x'(t) &= f(t, x(t), x(t-\tau), v(t), \quad t \in [0, t_f], \\ x(t) &= \phi(t), \quad t \in [-\tau, 0] \end{aligned}$$

where  $J$  is the objective function and  $L(\cdot)$  is Lagrangian objective function and  $V(t)$  is called an admissible control if and only if it fulfils the condition of  $a \leq v(t) \leq b$ ,  $t \in [0, t_f]$  with the set of admissible controls (admissible set or an admissible set of controls) denoted as  $V_{ad}$ . An efficient and effective numerical technique employing forward and backward approximation schemes to the adjoint system was use to solve the optimality problem and identify the best combination treatment strategy. Numerically results showed and surely confirmed and affirmed that indeed optimality treatment strategies can for sure decrease the loading of tumour cells and result in an increment of effector cells just after a few days of treatment.

(a) Re-write the excerpt to ensure consistency, conciseness and precision. [15 marks]

(b) State five (5) characteristics of a good **abstract of a thesis**. [5 marks]

**QUESTION B3 [20 Marks]**

B3 (a) Define the following terms as used in scientific writing

- i. *Theorem*.
- ii. *Plagiarism*.
- iii. *Hypothesis*.
- iv. *Abstract of a thesis*.
- v. *Nominalization*.

[15 marks]

(b) Consider the ODE system

$$\begin{aligned} \dot{x} &= -y + x^2, \\ \dot{y} &= -x + y^2. \end{aligned}$$

i. Does the system exhibit a unique solution? [3 marks]

ii. State the location of any fixed points of the system. [2 marks]

**QUESTION B4 [20 Marks]**

B4 (a) State any two componets of each of the following

- i. Title page of a dissertation.
- ii. Conclusion of a dissertation.
- iii. A bibtex file.
- iv. Introduction of a thesis.

[8 marks]

(b) Consider the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= 4x - y, \\ \frac{dy}{dt} &= 2x + y.\end{aligned}$$

- i. Determine the general solution of the system. [10 marks]
- ii. Give a plausible interpretation of the long term dynamics of the system. [2 marks]

**QUESTION B5 [20 Marks]**

B5 The number of aphids in a garden can be controlled by releasing ladybirds. The dynamics of aphids ( $A$ ) and ladybirds ( $L$ ) are given by the following ordinary differential equations:

$$\begin{aligned}\frac{dA}{dt} &= A(a - bA - cL), \\ \frac{dL}{dt} &= L(ecA - d).\end{aligned}$$

where  $a, b, c, d$  and  $e$  are positive constants.

- (a) Give ecological meanings of the parameters  $a, b, c$ , and  $d$ . [4 marks]
- (b) Determine the time invariant solutions of the aphids and ladybirds. [3 marks]
- (c) Which of the solutions in (b) above are stable? [5 marks]
- (d) Given that  $a = 2, b = 1, c = 1, d = 1, e = 1$ , sketch a plausible phase portrait of the model. [8 marks]

**QUESTION B6 [20 Marks]**

- B6 (a) State any five (5) advantages of typesetting documents with LaTeX. [5 marks]
- (b) Sipho is a fourth year student. He wishes to persue a masters degree in mathematics. Sipho is, nonetheless, not sure which field/area of mathematics he should specialize in. State, with very brief discussions, five (5) main determinants that Sipho could use to make a choice of the field of specialization. [10 marks]
- (c) Explain the difference between a *for loop* and a *while loop* as used in OCTAVE or MATLAB. [5 marks]