UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2019/2020

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Optimisation Theory

Course Number : MAT418

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B3, ..., B7) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Some formulas are given on the last page.

Special Requirements: NONE

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

Consider the following LP.

$$\begin{array}{llrcrcrcl} \max & z = & 3x_1 & + & 7x_2 & + & 5x_3 & & \\ \text{s.t.} & & x_1 & + & x_2 & + & x_3 & \leq & 50 \\ & & 2x_1 & + & 3x_2 & + & x_3 & \leq & 100 \\ & & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

(a) Find the dual of the LP. (5)

(b) Use the graphical method to solve the dual of the LP. (7)

(c) Use complementary slackness to solve the primal LP. (8)

QUESTION B4 [*20 Marks*]

(a) Find all local extrema and saddle points of the function

$$f(x_1, x_2) = x_1^4 + x_2^4 - 4x_1x_2 + 1.$$

(10)

(b) Use Golden Section Search to determine, within an interval of length 0.6, the optimal solution to

$$\begin{array}{lll} \max & z = & x - e^x \\ \text{s.t.} & & -1 \leq x \leq 3. \end{array}$$

(10)

QUESTION B5 [*20 Marks*]

Use the Kuhn-Tucker conditions to find the optimal solution to the following problem.

$$\begin{array}{lll} \min & z = & (x_1-4)^2 + (x_2-4)^2 \\ \text{s.t.} & & x_1 + x_2 \leq 4 \\ & & x_1 + 3x_2 \leq 9 \end{array}$$

QUESTION B6 [*20 Marks*]

(a) An entrepreneur wishes to sell hand sanitiser in Manzini and Mbabane. If x_1 emalangeni is spent on marketing in Manzini, then $4\sqrt{x_1}$ cases of hand sanitisers can be sold there, and if x_2 emalangeni is spent marketing in Mbabane, then $6\sqrt{x_2}$ cases can be sold there. Each case of sanitisers sold in Manzini sells for E90 and incurs E40 in production and transport costs. Each case of sanitisers sold in Mbabane sells for E100 and incurs E50 in production and transport costs. A total of E1000 is available for marketing. How much should the entrepreneur spend in marketing in each city in order maximise profits? (10)

(b) The Douglas-Cobb model says that when a company invests L units of labour and K units of capital, the production level P is given by

$$P = bL^{\alpha}K^{1-\alpha}$$

where $b > 0$ and $0 < \alpha < 1$ are constants. Suppose that the cost per unit labour is m emalangeni and the cost per unit capital is n emalangeni and that the company has a budget of B emalangeni to spend on total labour and capital. Show that maximum production occurs when

$$L = \frac{\alpha B}{m} \quad \text{and} \quad K = \frac{(1-\alpha)B}{n}.$$

(10)

QUESTION B7 [*20 Marks*]

(a) Use the method of steepest ascent to approximate the solution to

$$\begin{aligned} \max \quad z &= -(x_1-2)^2 - x_1 - x_2^2 \\ \text{s.t.} \quad & (x_1, x_2) \in \mathbb{R}^2. \end{aligned}$$

Start at the point $\left(\frac{5}{2}, \frac{3}{2}\right)$. (10)

(b) Perform *one* iteration of the feasible directions method on the following problem.

$$\begin{aligned} \max \quad & z = 2xy + 4x + 6y - 2x^2 - 2y^2 \\ \text{s.t.} \quad & x + y \leq 2 \\ & x, y \geq 0 \end{aligned}$$

Begin at the point $(0,0)$. (10)

END OF EXAMINATION PAPER