UNIVERSITY OF ESWATINI

RE-SIT EXAMINATION, 2019/2020

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Optimisation Theory

Course Number : MAT418

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B3, ..., B7) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Some formulas are given on the last page.

Special Requirements: NONE

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*20 Marks*]

(a) Give precise definitions of the following.

(i) Convex function from a convex set $S \subseteq \mathbb{R}^n$ to $\mathbb{R}$. (2)

(ii) Concave function from a convex set $S \subseteq \mathbb{R}^n$ to $\mathbb{R}$. (2)

(b) Determine whether the given function is a convex function, concave function or neither on $\mathbb{R}^2$. Explain.

(i) $f(x_1,x_2) = x_1^2 + 2x_1x_2 + x_2^2$. (5)

(ii) $f(x_1,x_2) = -x_1^2 - x_1x_2 - 2x_2^2$. (5)

(c) Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 2x^2 + x - 1$$

on the interval $[0,2]$. (6)

QUESTION A2 [*20 Marks*]

(a) Use the graphical method to solve the following LP.

$$\begin{array}{llrcrcl} \min & w = & 50y_1 & + & 100y_2 & & \\ \text{s.t.} & & 2y_1 & + & y_2 & \geq & 10 \\ & & y_1 & + & 3y_2 & \geq & 15 \\ & & & & y_1, y_2 & \geq & 0 \end{array}$$

(5)

(b) Find the dual of the following LP.

$$\begin{array}{llrcrcrcl} \min & w = & 4y_1 & + & 2y_2 & - & y_3 & & \\ \text{s.t.} & & y_1 & + & 2y_2 & & & \leq & 6 \\ & & y_1 & - & y_2 & + & 2y_3 & = & 8 \\ & & & & \multicolumn{5}{r}{y_1, y_2 \geq 0, y_3 \text{ urs}} \end{array}$$

(5)

(c) Use the Big-M method to solve the following LP.

$$\begin{array}{llrcrcl} \min & z = & 3x_1 & - & 4x_2 & & \\ \text{s.t.} & & x_1 & - & 2x_2 & \geq & 2 \\ & & x_1 & - & x_2 & \geq & 3 \\ & & & & x_1, x_2 & \geq & 0 \end{array}$$

(10)

END OF SECTION A – TURN OVER

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

Consider the following LP.

$$\begin{array}{llcrcrcrcl} \max & z & = & 4x_1 & + & x_2 & + & 2x_3 & & \\ \text{s.t.} & & & 8x_1 & + & 3x_2 & + & x_3 & \leq & 2 \\ & & & 6x_1 & + & x_2 & + & x_3 & \leq & 8 \\ & & & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

(a) Find the dual of the LP. (5)

(b) Use the graphical method to solve the dual of the LP. (7)

(c) Use complementary slackness to solve the primal LP. (8)

QUESTION B4 [*20 Marks*]

(a) Find all local extrema and saddle points of the function

$$f(x_1, x_2) = 3x_1x_2 - x_1^2x_2 - x_1x_2^2.$$

(10)

(b) Use Golden Section Search to determine, within an interval of length 0.3, the optimal solution to

$$\begin{array}{llcl} \max & z & = & -x^2 - 1 \\ \text{s.t.} & & & -0.5 \leq x \leq 1.5. \end{array}$$

(10)

QUESTION B5 [*20 Marks*]

Use the Kuhn-Tucker conditions to find the optimal solution to the following problem.

$$\begin{array}{llcl} \max & z & = & -(x_1 - 3)^2 - (x_2 - 5)^2 \\ \text{s.t.} & & & x_1 + x_2 \leq 7 \\ & & & x_2 \leq 4 \end{array}$$

QUESTION B6 [*20 Marks*]

(a) It costs E20 to purchase 1 hour of labour and E10 to purchase a unit of capital. If L hours of labour and K units of capital are available, then $L^{2/3}K^{1/3}$ machines can be produced. If E100 is available to purchase labour and capital, what is the maximum number of machines that can be produced? (10)

(b) Find the optimal solution to the following problem.

$$\begin{aligned} \max \ z &= -2x^2 - y^2 + xy + 8x + 3y \\ \text{s.t.} \quad & 3x + y = 10 \end{aligned}$$

(10)

QUESTION B7 [*20 Marks*]

(a) Use the method of steepest ascent to approximate the solution to

$$\begin{aligned} \max \ f(x_1, x_2) &= -(x_1 - 3)^2 - (x_2 - 2)^2 \\ \text{s.t.} \quad & (x_1, x_2) \in \mathbb{R}^2. \end{aligned}$$

Start at the point $(1,1)$. (10)

(b) Perform *one* iteration of the feasible directions method on the following problem.

$$\begin{aligned} \max \quad & z = 3xy - x^2 - y^2 \\ \text{s.t.} \quad & 3x + y \leq 4 \\ & x, y \geq 0 \end{aligned}$$

Begin at the point $(1,0)$. (10)

END OF EXAMINATION PAPER