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# UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2019/2020

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**BASS IV, B.Sc IV**

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**Title of Paper** : INTRODUCTION TO MATHEMATICS OF FINANCE

**Course Number** : MAT 442

**Time Allowed** : Three (3) Hours

**Instructions:**

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Start each new major question (A1-A5, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

**Special Requirements: NONE**

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

## SECTION A [40 Marks]: ANSWER ALL QUESTIONS

### QUESTION A1.

(a.) Define a bond. [2 marks]

(b.) What is the annual interest rate earned by a 53-day treasury bill with a maturity value (amount) of E120,000.00 that trades for E100,000.00 (principal) given 360 trading days in a year. [6 marks]

### QUESTION A2.

Suppose  $X : \Omega \rightarrow \mathfrak{R}$  is a given pricing function with cumulative distribution function (cdf),  $F(x) = P[X \leq x]$ . Show that  $X$  is a cadlag function. [8 marks]

### QUESTION A3.

A stock price  $Y$  for a given asset in trade changes according to the stochastic differential equation

$$dY_t = Y(t)dB_t; \quad Y(0) = \pi$$

Find the stock price [8 marks].

### QUESTION A4.

(a.) Define a Brownian Motion and give 2 financial industry-based Brownian processes. [4 marks]

(b.) Given a Brownian price process  $B(t)$ , show that the increments  $B_{t_1}, (B_{t_2} - B_{t_1}), (B_{t_3} - B_{t_2}), \dots$  at distinct times  $t_i; i = 1, 2, 3, \dots$  are independent. [4 marks]

### QUESTION A5.

An American call option expiring in 2-years has an exercise price of E300.00 on the Eswatini stock market and currently trades at E340.00. It is anticipated that the stock will rise by a factor of 1.25 and fall by a factor of 0.70. If the interest rate is 3%, find the value of the option. [8 marks]

## SECTION B: ANSWER ANY *THREE* QUESTIONS

### QUESTION B2

- (a.) Define a market. [4 marks]
- (b.) What is the price of a bond security whose face value  $F = \text{E}12,000.00$ , a coupon rate  $i$  of 12% and a time to maturity  $T$  of 45 years if the bond makes bi-annual interest payments and the rate of return on securities with same or similar characteristics (yield to maturity) is 5%. [13 marks]
- (c.) Differentiate a bond at premium and one at par. [3 marks]

### QUESTION B3.

- (a.) Define an Ito process. [3 marks]
- (b.) List three (3) properties of the Ito integral. [4 marks]
- (c.) Evaluate the integral  $I = \int_0^t B^3 dB(s)$ . [13 marks]

### QUESTION B4.

- (a.) Define an arbitrage market. [3 marks]
- (b.) Consider the market process  $X(t)$  given by

$$dX_1(t) = 3dt + 2dB_1(t)$$

$$dX_2(t) = -2dt + dB_1(t) + 5dB_2(t).$$

- Show that a portfolio  $\theta(t)$  should be allowed to do business in  $X(t)$ . [14 marks]
- (c.) Find the value process  $V^\theta(t)$  at expiration time  $t = T$ . [3 marks]

### QUESTION B5.

- (a.) An investor buys 1000 European call options on the Eswatini stock market with strike price  $E = \text{E}250.00$ . Given that the asset price at expiry  $S_T = \text{E}278.00$  and the premium  $c = \text{E}25.00$ , find the payoff for this option. [10 marks]
- (b.) Suppose at a given future time  $S_T < E$ . What is the payoff. [5 marks]
- (c.) A stock has an strike price of  $\text{E}40.00$ . If the stock price goes to  $\text{E}41.50$ , is the exchange likely to add a new strike price. [5 marks]

**QUESTION B6.**

(a.) Write the following markets  $X(t)$  in the form

$$dX(t) = u(t, \omega)dt + v(t, \omega)dB(t)$$

(i.)  $X(t) = tB^2(t)$ .

[4 marks]

(ii.)  $X(t) = 2 + 4t + e^{B(t)}$ .

[4 marks]

(b.) Find the solution to the Ornstein-Uhlenbeck equation

$$dS(t) = S(t)dt + dB(t); \quad S(0) = 10 \text{ units}$$

representing the change in price  $S(t)$  of an option trading  
in a stock market at time  $t \in [0, T]$ .

[12 marks]

**END OF EXAMINATION**