UNIVERSITY OF ESWATINI

MAIN EXAMINATION 1, 2019/2020

M.Sc. Mathematics 1

Title of Paper : FINANCIAL DERIVATIVES

Course Number : MAT 604

Time Allowed : Three (3) Hours

Instructions:

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Start each new major question (A1-A5, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1.

(a.) Define an Asset. [2 marks]

(b.) Give four (4) examples each of:

i.) Tangible asset. [4 marks]

ii.) Intangible asset. [4 marks]

QUESTION A2.

(a.) Define a financial derivative. [3 marks]

(b.) Identify the necessary reason for a financial derivative. [4 marks]

(c.) Give two (2) examples of financial derivatives. [3 marks]

QUESTION A3.

(a.) Define a call option contract. [2 marks]

(b.) Give three (3) types of option contracts with full description. [3 marks]

(c.) An investor buys 50,000 units of an European call option contract at a strike price E of $E80,000.00$. In 6-month time period the asset price $S_6 = E105,000.00$. Find the pay-off of this contract if the premium (c) is $E16,000.00$ [5 marks].

QUESTION A4.

(a.) Define a European (contingent) T-claim. [3 marks]

(b.) When is a T-claim attainable in an European financial market. [3 marks]

(c.) Define a portfolio in a market and give the condition for it to be self financed. [4 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2

(a.) Define a hedging portfolio for an European contract and state the necessary existence condition. [4 marks]

(b.) Mr. Dlamini takes a long (buy) position of one corn contract to buy 15,000 bushels for March 2020 delivery at a price of $E36.82$ per bushel at the Eswatini Board of Trade (EBOT). The contract requires maintenance margin of $E70,500.00$ with an initial margin markup of 160%, i.e. the initial margin which Mr. Dlamini and the seller each has to deposit into the broker's account on the first day they enter the contract. The next day the price of this contract drops to $E36.52$. The following day the price drops again to E32.52. Find the deposit amount that keeps the buyer in this contract if the mark-up is the hedging portfolio for this contract. [16 marks]

QUESTION B3

a.) Define an American option contract. [3 marks]

b.) An American call option derivative expiring in 3-years has an exercise price of E1500.00 on the Eswatini stock market and currently trades at E3840.00. It is anticipated that the stock will rise by a factor of 1.25 and fell by a factor of 1.00. If the interest rate is 9.9%, Find the upward price of the option at the third year and its pay-off. [17 marks]

QUESTION B4.

(a.) Define an admissible derivative. [4 marks]

(b.) Show that a portfolio $\theta(t)$ for an admissible derivative trading in an un-normalized market $X(t)$ is the same for the derivative in a normalized market. [16 marks]

QUESTION B5.

Suppose a derivative market such that $X_0(t) = e^{2\rho t}, \ \rho > 0$ and $X_1(t) \sim Y(t)$ where

$$dY(t) = \alpha Y(t)dt + \sigma dB(t); \ \ \rho, \alpha \in \Re$$

. How do we hedge the $T-$ claim $F(\omega) = e^{Y(t)}$. [20 marks]

QUESTION B6.

(a.) Find the price of a derivative whose changes follow the Black-Scholes equation

$dX(t) = \alpha X(t)dt + \rho X(t)dB(t); B(0) = 0, \alpha \in \Re.$ [$15 marks$]

(b.) Evaluate the Expected price if the expiration time of the derivative is at $T = 35$ years. [5 marks]

END OF EXAMINATION