UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2020/2021

BASS I, B.Ed I, B.Comm I

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Number : MAT 107

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate whether you are full time or part time student and indicate your program on your answer booklet.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) The polynomial $Ax^3+3x^2+Bx-12$ has $(x+3)$ as a factor. When the polynomial is divided by $(x+1)$ the remainder is -6. Find the values of A and B. [5]

b) Solve the exponential equation $10^{14x} = e^{6x+7}$ [5]

c) Prove that $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$. [5]

d) Find the fifth term in the expansion of $(2x - y)^8$. [5]

e) Using the method of mathematical induction, prove that

$$4 + 8 + 12 + \cdots + 4n = 2n(n + 1)$$

for all positive values of n. [5]

f) The geometric mean of p and 28 is 14. Find p. [5]

g) Solve the following linear system of equations using Crammer's rule. [5]

$$7x - 2y = -1$$
$$2x + 7y = 30$$

h) Find the equation of a straight line passing through the point $(-2, -1)$ and perpendicular to $6x - 4y = 8$. [5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Use synthetic division to find the quotient and remainder when

$$P(x) = 8x^2 - x^3 + 63$$

is divided by $D(x) = x - 4$. [8]

b) Find all the roots of the equation $6x^3 + 25x^2 + 3x - 4 = 0$. [12]

QUESTION B3 [*20 Marks*]

a) Solve the logarithmic equation [8]

$$\log_2(x - 5) + \log_2(x + 2) = 3.$$

b) $E11693$ is invested at 1.51% compounded monthly. After how many years will the investment exceed $E39000$? [12]

QUESTION B4 [*20 Marks*]

a) i) Find the first three terms of an arithmetic progression whose $9th$ term is 16 and $40th$ term is 47. [6]

ii) The first term of a geometric progression is 11 and the common ratio is 9. Find the sum of the first eight terms. [6]

b) Find the first three terms of the expansion of $(x^2 - 2y)^9$. [8]

QUESTION B5 *[20 Marks]*

a) Prove by mathematical induction that the following formula [8]

$$1(2) + 2(3) + 3(4) + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

is valid for all positive integers.

b) Solve the following linear system of equations using Crammer's rule. [12]

$$\begin{aligned} -x + 4y - z &= -3 \\ -3x + 2y + 2z &= -9 \\ 5x + 2y - 5z &= 13 \end{aligned}$$

QUESTION B6 *[20 Marks]*

a) If $v = \alpha\cos(\theta)$ and $w = \beta\sin(\theta)$, prove that $\frac{v^2}{\alpha^2} + \frac{w^2}{\beta^2} = 1$. [5]

b) Find the term that involves x^3 in the expansion of $\left(3x - \frac{1}{5x}\right)^9$. [5]

c) Find the center and radius of the circle $x^2 + y^2 - 2y - 48 = 0$. [5]

d) Express $(1+i)(2+3i)$ in the form $a + ib$. [5]

END OF EXAMINATION PAPER