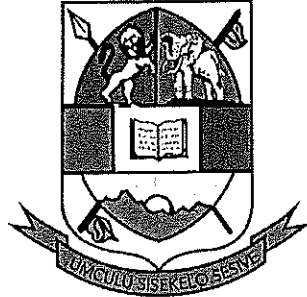# UNIVERSITY OF ESWATINI

MAIN EXAMINATION 2020/2021

BEng I, BSc I, BEd I, BSc IT I, BSc Comp Sci ED I, BASS I, BSc IS I, BSc IT-IDE I

Title of Paper   : ALGEBRA, TRIG. AND ANALYTIC GEOMETRY

Course Number   : MAT111

Time Allowed   : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.

2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.

3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.

4. Show all your working.

5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.

6. You can answer questions in any order.

7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1 (a)   i. State, without proof, the *rational root theorem*.   [2 marks]

ii. Use the *rational root theorem* to find all the possible roots of the polynomial
$P(x) = 2x^3 + x^2 - 13x + 6$.   [3 marks]

(b) Prove the identity: $\sec^2 x = \dfrac{\csc x}{\csc x - \sin x}$.   [5 marks]

(c) Evaluate $3i - (-2i + 1)^2 + 2$.   [5 marks]

(d) Given that the seventh term of an Arithmetic Progression is 41 while the fourteenth term is 77. Find the 20th term.   [5 marks]

(e) On the same axes, sketch the graphs of

i. $y = \log_7 x$.   [2 marks]

ii. $x^2 + y^2 = 4$.   [3 marks]

(f) Find the equation of a straight line passing through the points
$(-4, 8)$ and $(-3, -6)$.   [5 marks]

(g)   i. State any two properties of the dot product.   [2 marks]

ii. Find the angle between the vectors $A = -4\hat{i} + 5\hat{j} + 6\hat{k}$ and $B = -8\hat{j} - 5\hat{k}$, with your answer in degrees correct to 2 decimal places (d.p).   [3 marks]

(h) A sum of money E_0 Emalangeni is invested in an account that pays interest at $r\%$ p.a. compounded daily. After t years, the amount in the account is given by

$$h(t) = E_0 \left(\frac{365 + r}{365} \right)^{365t} .$$

i. If the amount of the account doubles in 3 years, find r.   [3 marks]

ii. What time would it take for the amount on the account to become $10E_0$?   [2 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 Given the following matrices

$$A = \begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -2 & 4 \\ 2 & 1 & 5 \end{pmatrix} \text{ and } C = \begin{pmatrix} 20 & 101 & 31 \\ 0 & -1 & 1 \\ 1 & 1 & 10 \end{pmatrix}.$$

(a) Compute the following

    i. $6A - 4B^T$ [3 marks]

    ii. $C + AB$ [7 marks]

    iii. $|C|$ [5 marks]

(b) Find the sum of the geometric progression (G.P) $8, -4, 2, \cdots \frac{1}{128}$.   [5 marks]

QUESTION B3 [*20 Marks*]

B3 (a) Use mathematical induction to prove that for all integers $n \geq 1$,

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2.$$

                                                                          [8 marks]

  (b) Consider the binomial expnsion of

$$\left(x^3 - \frac{y^4}{x^2} \right)^{20}.$$

    Find

      i. the first three terms   [3 marks]

      ii. the term involving x^{-20} [4 marks]

  (c) Use synthetic division to determine the quotient and remainder of

$$\frac{2x^4 - 4x + 2x^2 - 9}{2x - 2}.$$

                                                                         [5 marks]

QUESTION B4 [*20 Marks*]

B4   (a) Use Cramer's rule to solve the system

$$x + y + z = 6$$
$$3x - 2y + z = 2$$
$$x + 2y + 3z = 14.$$

[12 marks]

(b) Find the particular solution of

$$2\sin^2\theta - 3\sin\theta + 1 = 0$$

in radians, in the interval $0 < \theta < 2\pi$.   [8 marks]

QUESTION B5 [*20 Marks*]

B5   (a) Use Demoivre's theorem to evaluate

$$(1 - 2i)^{12}$$

expressing your answer in the form $a + bi$.   [8 marks]

(b) A circle is centred on the straight line $2x + 5y + 1 = 0$ and passes through the points $(2, 3)$ and $(-1, 6)$.
   i. Find the equation of the circle.   [7 marks]
   ii. Find the equation of the tangent of the circle at $(-1, 6)$.   [5 marks]

QUESTION B6 [*20 Marks*]

B6   (a) Given that $\cos A = -0.5$ and A lies in the second quadrant, find the exact values of
   i. $\sin 2A$ [2 marks]
   ii. $\tan 2A$ [5 marks]

(b) Find the value of

$$\sum_{n=0}^{200}(7n + 9)$$

[6 marks]

(c) FInd the area of a triangle whose vertices are given by
   $X(6, -2, 2)$, $Y(10, 8, -3)$ and $Z(-7, 5, 6)$.   [7 marks]

END OF EXAMINATION