UNIVERSITY OF ESWATINI

RE-SIT EXAMINATION, 2020/2021

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Optimisation Theory

Course Number : MAT418

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B3, ..., B7) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Some formulas are given on the last page.

Special Requirements: NONE

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [20 Marks]

(a) Give the precise definition a convex function from a convex set $S \subseteq \mathbb{R}^n$ to $\mathbb{R}$. (2)

(b) Show that for $c \leq 0$, if f is a convex function on a convex set S, the function cf is concave on S. (3)

(c) For each function below, determine whether it is convex, concave, or neither on $\mathbb{R}^2$.

(i) $f(x_1, x_2) = x_1^2 + 3x_1x_2 + x_2^2$. (5)

(ii) $f(x_1, x_2) = -x_1^2 - x_1x_2 - x_2^2$. (5)

(iii) $f(x_1, x_2) = x_1^2 + x_2^2 - 2x_1 + 4x_2 + 5$. (5)

QUESTION A2 [20 Marks]

(a) Consider the following linear programming problem.

$$\begin{array}{lrcrcrl}
\text{maximise} & z & = & 3x_1 & + & x_2 & \\
\text{subject to} & & & x_1 & + & x_2 & \geq 3 \\
& & & 2x_1 & + & x_2 & \leq 4 \\
& & & x_1 & + & x_2 & = 4 \\
& & & & & x_1, x_2 & \geq 0
\end{array}$$

Write down the dual of the problem without first converting it to normal form (use the rules). (6)

(b) Consider the following linear programming problem.

$$\begin{array}{lrcrcrl}
\text{minimise} & z & = & 2x_1 & + & 3x_2 & \\
\text{subject to:} & & & 2x_1 & + & x_2 & \geq 4 \\
& & & -x_1 & + & x_2 & \leq 1 \\
& & & & & x_1, x_2 & \geq 0
\end{array}$$

(i) Use the graphical method to solve the problem. (6)

(ii) Use the Big-M method to solve the problem. (8)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [20 Marks]

Consider the following linear programming problem.

$$\begin{array}{llrcrcrcl} \max & z = & 4x_1 & + & x_2 & + & 2x_3 & & \\ \text{s.t.} & & 8x_1 & + & 3x_2 & + & x_3 & \leq & 2 \\ & & 6x_1 & + & x_2 & + & x_3 & \leq & 8 \\ & & & & & x_1, & x_2, x_3 & \geq & 0 \end{array}$$

(a) Find the dual of the problem. (5)

(b) Use the graphical method to solve the dual of the LP. (7)

(c) Use complementary slackness to solve the primal problem. (8)

QUESTION B4 [20 Marks]

(a) Find all local extrema and saddle points of the function

$$f(x_1, x_2) = x_1^3 - 3x_1x_2^2 + x_2^4.$$

(10)

(b) Use Golden Section Search to determine, within an interval of length 0.2, the optimal solution to

$$\begin{array}{rl} \text{maximise} & f(x) = 2 - (x-1)^2 \\ \text{subject to:} & 0.75 \leq x \leq 1.25. \end{array}$$

(10)

QUESTION B5 [20 Marks]

Use the Kuhn-Tucker conditions to find the optimal solution to the following problem.

$$\begin{array}{lrl} \text{maximise} & z = & x_1(30 - x_1) + x_2(50 - 2x_2) - 3x_1 - 5x_2 - 10x_3 \\ \text{subject to:} & & x_1 + x_2 - x_3 \leq 0 \\ & & x_3 \leq 18. \end{array}$$

QUESTION B6 [20 Marks]

(a) It costs E20 to purchase 1 hour of labour and E10 to purchase a unit of capital. If L hours of labour and K units of capital are available, then $L^{2/3}K^{1/3}$ machines can be produced. If E100 is available to purchase labour and capital, what is the maximum number of machines that can be produced? (10)

(b) Find the optimal solution to the following problem.

$$\begin{array}{llcl} \max & z & = & x_1^2 + 2x_2^2 \\ \text{s.t.} & & & x_1^2 + x_2^2 = 1. \end{array}$$

(10)

QUESTION B7 [20 Marks]

(a) Use the method of steepest ascent to approximate the solution to

$$\begin{array}{lll} \text{maximise} & f(x_1, x_2) = & -(x_1-2)^2 - x_1 - x_2^2 \\ \text{subject to:} & & (x_1, x_2) \in \mathbb{R}^2. \end{array}$$

Start at the point $\left(\frac{5}{2}, \frac{3}{2}\right)$. (10)

(b) Perform *one* iteration of the feasible directions method on the following problem.

$$\begin{array}{ll} \text{maximise} & f(x_1, x_2) = 3x_1x_2 - x_1^2 - x_2^2 \\ \text{subject to:} & 3x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{array}$$

Begin at the point $(1, 0)$. (10)

END OF EXAMINATION PAPER