UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2020/2021

BSc IV, BASS IV

Title of Paper : INTRODUCTION TO MATHEMATICS OF FINANCE

Course Number : MAT442

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1 – A3, B4 – B8) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: Statistical Tables

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1 (a) Outline any five operations of financial institutions that help sustain their businesses. [5 marks]

(b) Name any two risks that banks face in their operations and suggest possible ways to mitigate them. [4 marks]

A2 (a) State the σ - algebra theorem. [5 marks]

(b) Let Ω denote the set of all outcomes when tossing an unbiased coin 3 times. Describe the probability space. [8 marks]

(c) Company XYZ bonds pay interest semi annually and mature in 10 years. Currently, a E1000 bond sells for E800 and the bondholders require annual return of 9%. Calculate the coupon rate of these bonds. [8 marks]

A3 (a) State the Ito's formula. [2 marks]

(b) Using Ito's formula, express the following Ito integral in terms of a standard integral of Brownian motion,

$$\int_0^t B_s^2 dB_s$$

. [8 marks]

SECTION B [*60 Marks*]: ANSWER ANY *THREE* QUESTIONS

QUESTION B4 [*20 Marks*]

B4 (a) Define a put-call parity. [4 marks]

(b) Using the Black Scholes model, determine the price of a European call option on a non-dividend paying stock, where the stock price is $E630$, the strike is $E600$, the time to expiry is 6 months, the risk-free rate is 10% and the volatility is 20%. [16 marks]

QUESTION B5 [*20 Marks*]

B5 (a) A stock price Y for a given asset in trade changes according to the stochastic differential equation

$$dY(t) = \frac{\pi}{4}Y(t)dB_t, Y(0) = \pi^2.$$

Find the stock price. [10 marks]

(b) Use the Ito's formula to write $X(t) = tB^2(t)$ in the form

$$dX(t) = u(t,\omega)dt + v(t,\omega)dB(t).$$

[10 marks]

QUESTION B6 [*20 Marks*]

B6 A stock price is currently $E1500$. Over each of the next three six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum.

(a) Construct a stock price tree. [5 marks]

(b) Using the tree, what is the value of the European call option with a strike price of $E1500$? [5 marks]

(c) Calculate the risk neutral probabilities. [10 marks]

QUESTION B7 [*20 Marks*]

B7 The Ornstern-Uhlenbeck process can be defined as the solution to the SDE,

$$dX_t = -\alpha X_t dt + \sigma dB_t,$$

$$X_0 = x_0.$$

(a) Apply the Ito's formula to solve the SDE. [17 marks]

(b) State the distribution of the process Xt. [3 marks]

QUESTION B8 [*20 Marks*]

B8 (a) State the Martingale representation theorem. [5 marks]

(b) Consider a simple model with $T = 2$ and $K = 4$. Suppose $r = 0$ and the risky security is as follows:

ω_k	t = 0	t = 1	t = 2
ω_1	$S_0 = 10$	$S_1 = 12$	$S_2 = 14$
ω_2	$S_0 = 10$	$S_1 = 12$	$S_2 = 10$
ω_3	$S_0 = 10$	$S_1 = 8$	$S_2 = 10$
ω_4	$S_0 = 10$	$S_1 = 8$	$S_2 = 6$

Calculate the discrete time martingale measure Q. [15 marks]

END OF EXAMINATION