UNIVERSITY OF ESWATINI

RESIT/SUPPLEMENTARY EXAMINATION, 2020/2021

BSc IV, BASS IV

Title of Paper : INTRODUCTION TO MATHEMATICS OF FINANCE

Course Number : MAT442

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1 – A3, B4 – B8) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: Statistical Tables

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1 (a) Distinguish between a stock and a bond, giving examples. [4 marks]

(b) Define an arbitrage market. [3 marks]

(c) Give two examples of martingales. [2 marks]

A2 (a) State the σ - algebra theorem. [5 marks]

(b) Consider a stock price behaviour in which the stock price either goes up by a factor 1.2 with probability 0.6 or goes down by a factor 0.6 with probability 0.4, over a time period $[t_0, t_1]$. Describe the probability space. [8 marks]

(c) What is the price p of a bond that makes semi annual coupon payments with face value of $E4850$, a coupon rate i of 3.75%, a time to maturity T of 20 years and the rate of return on securities with similar characteristics (yield to maturity) is 10%. [8 marks]

A3 (a) Use the Ito's formula to write $X(t) = t^3 + \exp B(t)$ in the form $dX(t) = u(t, \omega)dt + v(t, \omega)dB_t$. [10 marks]

SECTION B [*60 Marks*]: ANSWER ANY *THREE* QUESTIONS

QUESTION B4 [*20 Marks*]

B4 (a) State the Black Scholes put option formula, defining all its variables. [5 marks]

(b) Using the Black Scholes model, determine the price of a European put option on a non-dividend paying stock, where the stock price is $E450$, the strike is $E330$, the time to expiry is 6 months, the risk-free rate is 10% and the volatility is 20%. [15 marks]

QUESTION B5 [*20 Marks*]

B5 (a) State the Ito's formula. [4 marks]

(b) Evaluate the integral $I = \int_0^t t^2 B^2(s) dB(s)$. [16 marks]

QUESTION B6 [*20 Marks*]

B6 A stock price is currently $E1000$. Over each of the next three six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 6% per annum.

(a) Construct a stock price tree. [5 marks]

(b) Using the tree, what is the value of the European call option with a strike price of $E1000$? [5 marks]

(c) Calculate the risk neutral probabilities. [10 marks]

QUESTION B7 [*20 Marks*]

B7 Find the solution to the Ornstern-Uhlenbeck equation,

$$dS_t = 8S_t dt + 0.25dB_t,$$
$$S_0 = 15units.$$

representing the change in price S_t of an option trading in an African stock market at time $t \in [0, t]$. [20 marks]

QUESTION B8 [*20 Marks*]

1. What is a complete market. [5 marks]
2. Consider a simple model with $T = 2$ and $K = 4$. Suppose $r = 0$ and the risky security is as follows:

ω_k	t = 0	t = 1	t = 2
ω_1	$S_0 = 100$	$S_1 = 125$	$S_2 = 145$
ω_2	$S_0 = 100$	$S_1 = 125$	$S_2 = 115$
ω_3	$S_0 = 100$	$S_1 = 85$	$S_2 = 115$
ω_4	$S_0 = 100$	$S_1 = 85$	$S_2 = 70$

Calculate the discrete time martingale measure Q. [15 marks]

END OF EXAMINATION