UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2020/2021

B.Sc IV

Title of Paper : DYNAMICS II

Course Number : MAT 455

Time Allowed : Three (3) Hours

Instructions:

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Start each new major question (A1-A5, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1.

(a.) State Newton's first law of motion. [2 marks]

(b.) What is the final velocity v of a body with an initial velocity u of $2m/s$ accelerating at a rate a of $4m/s^2$ in time t of $4s$. [6 marks]

QUESTION A2.

(a.) Define an Inertial frame. [2 marks]

(b.) Design a 2-ideal and isolated system. Put a particle of mass m_1 on the first frame and derive the expression for its position, velocity and acceleration on the 2nd inertial frame if the velocity of the first frame is constant. [6 marks]

QUESTION A3.

(a.) State the law of conservation of linear momentum. [2 marks]

(b.) Show that the angular momentum vector L of a particle subject to no torque N is conserved. [6 marks]

QUESTION A4.

(a.) Define the *Langrangian* L of a body of mass m along the Y axis. [4 marks]

(b.) How did your definition in (A4a) above differs in case the motion is along the XYZ axes. [4 marks]

QUESTION A5.

(a.) Define the following terms:

(i.) Generalized coordinates. [2 marks]

(ii.) Degree of freedom. [1 marks]

(iii.) Constraint. [2 marks]

(b.) An arbitrary object placed on a 3 dimensional surface is described by the cartesian coordinates (2,0,1) and (3,1,1). Find the size of holonomic constraints on this object. [3 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2

(a.) State Hamilton's variational principle. [3 marks]

(b.) Show that the Langrange's equation of motion for a one dimensional oscillator without damping is

$m\ddot{x} + kx = 0.$ [7 *marks*]

(c.) Find the equation of motion of a simple pendulum of mass m and length l on the XYZ plane onto the polar plane. [10 marks]

QUESTION B3.

(a.) Define the *Hamiltonian* H. [2 marks]

(b.) What is the difference between the Hamiltonian H and the Langrangian L of a body. [3 marks]

(c.) A projectile of mass m moving with a velocity $\dot{x}$ traveled a vertical distance x_2. Show that the Hamiltonian H is given by:

$$H(p_i, x_i, t) = \frac{1}{2m}\sum_i p_i^2 + mgx_2.$$ [15*marks*]

QUESTION B4.

(a.) What is a *scleronomic* constraint. [3 marks]

(b.) Show that the Hamiltonian H of a body of mass m moving horizontally with a velocity v respective to a launch point, a height y and a distance d from the launch point as the generalized coordinates is given by:

$$H = p_d\left[\frac{p_d}{2m} - v\right] + \frac{p_y^2}{2m} + mgy.$$ [17*marks*]

QUESTION B5.

(a.) Consider an object of mass m falling near earth in 1D motion. What is its Langrangian [17 marks]

(b.) A student throws 5 objects on a 3 -dimensional surface. Find the degrees of freedom associated with the throw. Draw the freedom space for

the motion of the objects. [3 marks]

QUESTION B6.

(a.) What are *cyclic* coordinates. [3 marks]

(b.) State and Prove the Hamiltonian equations of motion. [17 marks]

END OF EXAMINATION