University of Eswatini

Final Examination, October 2021

B.Sc IV, BASS IV

Title of Paper : Fluid Mechanics

Course Code : MAT456/M455

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) State the four assumptions required in Bernoulli's equation. [4]

(ii) Describe Ludwig Prandtl boundary layer hypothesis. [4]

(iii) Name four physical quantities that are conserved and two quantities that are not conserved during a process. [6]

(b) (i) Find the streamlines through the point (x_0, y_0) for the following two-dimensional flow field:

$U = ay\hat{i} + bt\hat{j}$, where a, b are positive constants. [5]

(ii) Consider the following steady, two dimensional velocity field

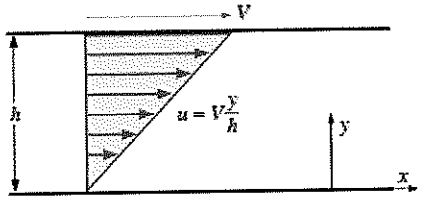
$$\vec{V} = (0.66 + 2.1x)\hat{i} + (-2.7 - 2.1y)\hat{j}$$

Is there a stagnation point in this flow field? If so, where it is? [5]

(iii) Consider fully developed Couette flow—flow between two infinite parallel plates separated by distance h, with the top plate moving and the bottom plate stationary as illustrated in the figure. The flow is steady, incompressible, and two-dimensional in the xy-plane. The velocity field is given by

$$\vec{V} = V\frac{y}{h}\hat{i} + o\hat{j}$$

Is this flow rotational or irrotational? If it is rotational, calculate the vorticity component in the z-direction. Do fluid particles in this flow rotate clockwise or counterclockwise? [6]



(iv) Find the equations of the pathlines for a fluid flow with velocity field $\mathbf{u} = ay\hat{i} + bt\hat{j}$, where a, b are positive constants.

Sketch the pathlines of the fluid particles which pass through the points $(X, 0)$ at time $t = 0$, for $X = -1, 0, 1, 2, 3$. [7]

(v) Express the Bernoulli equation in three different ways using (a) energies, (b) pressures, and (c) heads. [3]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) A velocity potential for a two dimensional flow is given by $\phi = (x^2 - y^2) + 3xy$. Calculate the stream function. [10]

(b) The velocity along the centerline of a nozzle of length L is given by

$$V = 2t\left(1 - \frac{x}{2L}\right)^2$$

where V is velocity in m/s and t is time in seconds from the commencement of the flow, x is distance from the inlet to the nozzle. Find the convective acceleration, local acceleration and total acceleration when $t = 3s$, $x = 0.5m$ and $L = 0.8m$. [10]

Question 3

(a) Determine the pathline and the streamline which pass through the origin at $t = 0$ for the velocity field $u = a\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j}$, where a, ω are positive constants. [10]

(b) Determine the acceleration of a fluid particle when velocity distribution is

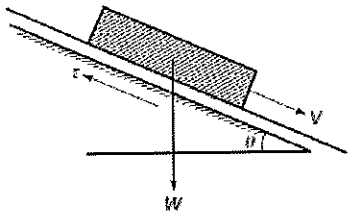
$$\mathbf{q}=\mathbf{i}(Ax^2yt)+\mathbf{j}(By^2zt)+\mathbf{k}(Czt^2)$$

where A, B and C are constants. Also find the vorticity components. [10]

Question 4

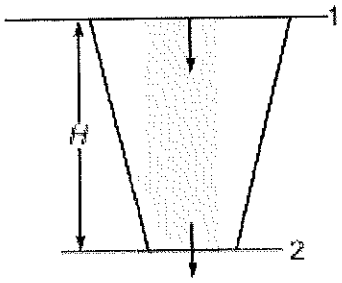
(a) The velocity v of a fluid beyond which streamline flow cease and turbulence begins depend the radius r of the tube, density ρ and viscosity μ of the fluid. Using dimensions obtain an expression which relates v, ρ, r and μ. [10]

(b) A 90 N rectangular solid block slides down a 30^0 inclined plane. The plane is lubricated by a 3 mm thick film of oil of relative density of 0.90 and viscosity 8.0 poise. If the contact area is 0.3 m^2, estimate the terminal velocity of the block (Refer the figure). [10]



Question 5

(a) A liquid flows downward through a tapered vertical pipe as shown in the figure. The pressure at section 1 and 2 are equal and the vertical distance between sections 1 and 2 is H. The diameter at section 1 is twice that at section 2. If $H = 1.5m$, find the velocities at sections 1 and 2 by assuming zero loss of energy between the two sections. [10]



(b) Consider the steady, two-dimensional, incompressible velocity field of $\vec{V} = (ax + b)\hat{i} + (-ay + cx)\hat{j}$. Calculate the pressure as a function of x and y. Assume gravity does not act in either c- or y-directions. [10]

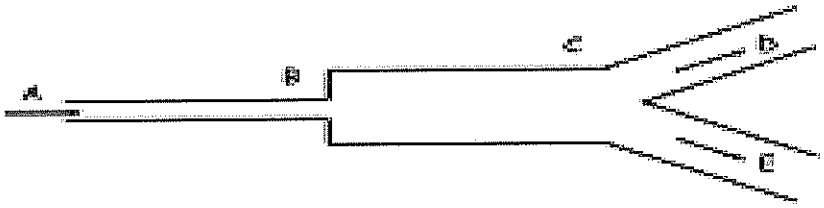
Question 6

(a) Write the nondimensional form of the equation

$$\frac{N(t)}{dt} = r_0 N(t), \quad t > 0, \quad N(0) = N_0 > 0.$$ [8]

(b) Water flows through a pipe AB (as shown in the figure) of diameter $d_1 = 50mm$, which is in series with pipe BC of diameter $d_2 = 75mm$ in which the mean velocity $v_2 = 2m/s$.

At C the pipe forks and one branch CD is of diameter d_3 such that the mean velocity $V_3 = 1.5m/s$. The other branch CE of diameter $d_4 = 30mm$ and conditions are such that the discharge Q_2 from BC divides so that $Q_4 = 0.5Q_3$. Calculate the values of $Q_1, V_1, Q_2, Q_3, d_3, Q_4$ and V_4 . [12]



End of Examination Paper