UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2020/2021

M.Sc. Mathematical Modelling & Mathematical Finance

Title of Paper : Research Methods

Course Number : MAT602

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions.
2. Answer ANY FIVE (5) questions.
3. Show all your working.
4. Start each new major question (1, 2 – 6) on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1 [*20 Marks*]

Write a Latex script that could be used to produce the following. It should include the preamble and all needed packages. The graph is saved as capture.png. The font of the words is 12pt. [20]

1 Results

1.1 Graphs

Graphs can have one vertex or many, and may or may not have edges [1]. Below is an example of a graph. While a graph is an abstract object, the vertices and edges often represent something, for example town and roads connecting them

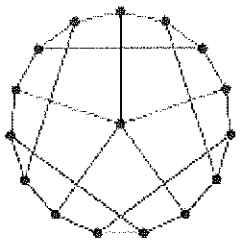

Figure 1: graph enumeration

1.2 Table

In Table 1, the bivariate Chebyshev spectral collocation quasilinearization method for nonlinear evolution parabolic equations was introduced for the first time.

$t \setminus N_x$	4	6	8
0.2	1.986e-008	1.119e-011	7.171e-013
0.6	5.577e-008	4.864e-011	1.005e-012

Table 1: Maximum errors E_N for the Cahn-Hillard equation, with $N_t = 10$

References

[1] Herzberg, A.M. and Murty, M.R., 2007. Sudoku squares and chromatic polynomials. Notices of the AMS, 54(6), pp.708-717.

QUESTION 2 [*20 Marks*]

Write the following using Latex commands

a) The big matrix A is given by $A = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \xi_{21} & \xi_{22} & \xi_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix}$. [7]

b) Consider the 1-D parabolic differential equation [8]

$$u_t = u_{xx}, \quad (x,t) \in (0,1) \times (0,T)$$

c) This is a system of differential equations [5]

$$\frac{dx}{dt} = f(t,x,y)$$
$$\frac{dy}{dt} = g(t,x,y).$$

QUESTION 3 [*20 Marks*]

a) Write down a scheme that could be used to solve the following system of differential equations

$$\frac{dx_1}{dt} = f_1(t,x_1,x_2,x_3), \quad x_1(0) = \beta_1$$
$$\frac{dx_2}{dt} = f_2(t,x_1,x_2,x_3), \quad x_2(0) = \beta_2$$
$$\frac{dx_3}{dt} = f_3(t,x_1,x_2,x_3), \quad x_3(0) = \beta_3.$$

using RK-2 method. [8]

b) Write down a pseudocode that could be used to implement the scheme in a). [12]

QUESTION 4 [*20 Marks*]

a) Consider the boundary value problem

$$a_2u''(x) + a_1u'(x) + a_0u(x) = f(x), \quad c < x < d$$

subject to

$$u(c) = \alpha, \quad u(d) = \beta.$$

Using the centered difference approximation, determine the finite difference scheme that can be used to solve the boundary value problem. Express your scheme in matrix form indicating the implementation of the boundary conditions. [8]

b) Write down a pseudocode that could be used to implement the scheme in a). [12]

QUESTION 5 [*20 Marks*]

a) Consider the Cauchy problem for the wave equation with $-\infty < x < \infty$ and $t > 0$: [8]

$$\rho_{tt} = v^2\rho_{xx},$$
$$\rho(x,0) = \phi(x),$$
$$\rho_t(x,0) = \psi(x),$$

where v is a constant. Using finite differences, determine a scheme for the wave equation.

b) Write down a pseudocode that could be used to implement the scheme in a). [12]

QUESTION 6 [*20 Marks*]

a) An effective introduction for a paper exhibits the four components that readers find useful as they begin to read a paper. State and discuss these four components and give examples where necessary. [12]

b) In any effective abstract, any scientist reading it should be able to understand why the work was carried out and why it is important (context and need), what the authors did (task) and what the paper reports about this work (object of the document), what the authors found (findings), what these findings mean (the conclusion), and possibly what the next steps are (perspectives).Consider the following abstract written in italic form. Discuss if the abstract is an effective one. [8]

Cancer treatment is an inexact science despite traditional cancer therapies. The traditional cancer treatments have high levels of toxicity and relatively low efficacy. Current research and clinical trials have indicated that virotherapy, a procedure which uses replication-competent viruses to kill cancer cells, has less toxicity and a high efficacy. However, the interaction dynamics of the tumor host, the virus, and the immune response is poorly understood due to its complexity. We present a mathematical analysis of models that study tumor-immune-virus interactions in the form of differential equations with spatial effects. A stability analysis is presented and we obtained analytical traveling wave solutions. Numerical simulations were obtained using fourth order Runge-Kutta and Crank-Nicholson methods. We show that the use of viruses as a cancer treatment can reduce the tumor cell concentration to a very low cancer dormant steady state or possibly deplete all tumor cells in body tissue. The traveling waves indicated an exponential increase and decrease in the cytotoxic-T-lymphocytes (CTLs) density and tumor load in the long term respectively.

QUESTION 7 [*20 Marks*]

a) Consider the following introduction and then answer the questions that follow:

Introduction

Bovine tuberculosis (BTB) results from infection by mycobacterium bovis (M. bovis), a Gram positive, acid-fast bacterium in the mycobacterium tuberculosis complex of the family mycobacteriaceae [2]. BTB is a chronic bacterial disease that affects all species of mammals including buffalo, cattle and humans [1], [2]. It is spread through inhaling contaminated aerosol droplets, drinking unpasteurised milk, eating poorly cooked meat, scavenging infected animals, grazing on contaminated pasture and also through vertical transmission (from an infected female animal to a newly born animal) [3], [4], [13], [14], [15], [16], [17]. BTB infection can be dormant for years and reactivate later in the lifespan of an animal due to stress or old age [2], [3], [14], [18].

It has been estimated that M. bovis accounts globally for 3.1% of all human tuberculosis cases (2.1% of all pulmonary and 9.4% of all extra-pulmonary tuberculosis cases). However, the extent of M. bovis involvement in the global tuberculosis burden in Africa is still largely unknown. This can be partly explained by the fact that in humans, tuberculosis due to M. bovis is indistinguishable from that due to M. tuberculosis in terms of clinical signs, radiological and pathological features. In addition, various laboratories in sub-Saharan Africa do not have the capacity to differentiate M. bovis from M. tuberculosis [20]. The paucity of information on BTB infection in Africa [5] with the exception of South Africa where a substantial research in BTB infection has been carried [14], has led to a poor understanding of its transmission dynamics in animals and humans. This work aims to enhance

the understanding of the transmission dynamics of BTB infection through mathematical models.

Mathematical modelling has become an important tool in analyzing the epidemiological characteristics of infectious diseases and can provide insight into useful control measures [21]. Various models have been formulated to explore various aspects of BTB infection. A non-linear transmission model consisting of susceptible and infected possum populations was developed [22]. The model was used to explain bovine tuberculosis dynamics in a heterogeneous possum population, taking into account the patchy distribution of the infection. A deterministic/stochastic model was developed to explore the factors that drive the spread of BTB infection in possum population and social contact was found to promote the spread of BTB infection [6]. A spatial stochastic model was developed in [23] to assess fertility control as a means of controlling bovine tuberculosis in badgers. The results showed that fertility control alone cannot completely eradicate BTB infection from badger populations.

Apart from using mathematical models to understand the transmission dynamics of BTB infection in wildlife, epidemiological models were also used in livestock. A model comprised of seven sub-populations was formulated to enhance the understanding of the transmission dynamics in cattle [4]. One of the main results of the model was that imported infected cattle were responsible for the persistence of BTB infection in cattle population. A discrete mathematical model was developed in [14] to assess vaccination as a control strategy in an ongoing epidemic of bovine tuberculosis in African buffalo. The model established that BTB infection can be completely wiped out if various strategies are used in combination. The concept of cross-infection of BTB between buffalo and cattle populations was not considered in any of the models formulated so far. This is in contrast to other zoonotic diseases like brucellosis disease that is transmitted from sheep to people [24]. The inclusion of cross-infection transmission route in the model was necessitated by the observation that BTB infection prevalence was higher in cattle and buffalo populations, in areas at the interface with wildlife than those that are not at the interface [7]. This suggested that cross-infection route may play a role in the epidemiology of BTB infections in both populations. It is at the interface where cattle/buffalo interactions are experienced [13]. We will build a model that will characterize the epidemiological features of BTB infection transmission mechanisms involving buffalo and cattle populations. The model will address the following questions:

- Does the shedding off of M. bovis in the environment promote the persistence of the infection in cattle and buffalo populations?
- Is cross- infection of BTB infection from buffalo responsible for higher prevalence of the infection in cattle?

i) What are the research questions being addressed by this article? [6]

ii) What are the main contributions by the authors to the current available literature? [4]

iii) Did the authors stated the need for their work? If so, state the need for their work. [4]

b) Express the following Harvard refences in APA style

i) Mkhatshwa, M.P., Motsa, S.S., Ayano, M.S. and Sibanda, P., 2020. MHD mixed convective nanofluid flow about a vertical slender cylinder using overlapping multi-domain spectral collocation approach. Case Studies in Thermal Engineering, 18, p.100598. [3]

ii) Kameswaran, P.K., Makukula, Z.G., Sibanda, P., Motsa, S.S. and Murthy, P.V.S.N., 2014. A new algorithm for internal heat generation in nanofluid flow due to a stretching sheet in a porous medium. International Journal of Numerical Methods for Heat & Fluid Flow. [3]

END OF EXAMINATION PAPER