UNIVERSITY OF ESWATINI

MAIN EXAMINATION I, 2020/2021

M.Sc. Mathematics I

Title of Paper : FINANCIAL DERIVATIVES

Course Number : MAT 604

Time Allowed : Three (3) Hours

Instructions:

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Start each new major question (A1-A4, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1.

Design a Mark-up derivative contract using the following pieces of information.

Bongekile (writer), Kunene (Long position), derivative contract ($1000 sticks/month$), delivery price ($E9.00$), Maintenance margin ($m = E8000$), Mark-up ($M = 150\%\ of\ m$), first month of contract ($price/corn = E8.00$), Second month of contract ($E7.90$), third month of contract ($E7.50$), Hedging ($H = M + m$)

(i.) Tabulate the contract flow for these months.

(ii.) State the conditions that keep Kunene in this contract after the third month. [10 marks]

QUESTION A2.

(a.) Define a barrier option. [2 marks]

(b.) Identify 2 reasons for a financial derivative. [4 marks]

(c.) Give 2 differences between forwards and futures. [4 marks]

QUESTION A3.

(a.) Define an Asian option contract. [3 marks]

(b.) An investor buys 10,000 units of an European call option contract at a strike price E of $E8000.00$. In 12-month time period the asset price $S_{12} = E9,000.00$. Find the pay-off of this contract if the premium is $E600.00$ [7 marks].

QUESTION A4.

(a.) Define a pay-off. [3 marks]

(b.) When is a T-claim attainable in an European financial market. [3 marks]

(c.) Define a portfolio in a market and give the condition for it to be self financed. [4 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2

(a.) Define a hedging portfolio for an European contract and state the necessary existence condition. [4 marks]

(b.) Mr. Motsa takes a long (buy) position of a chicken contract to buy 20,000 chicks for December 2021 delivery at a price of $E16.82$ per chick at the Eswatini Board of Trade (EBOT). The contract requires maintenance margin of $E20,500.00$ with an initial margin markup of 110%, i.e. the initial margin which Mr. Motsa and the seller each has to deposit into the broker's account on the first day they enter the contract. The next day the price of this contract drops to $E16.52$. The following day the price drops again to E16.50. Find the deposit amount that keeps the buyer in this contract if the mark-up is the hedging portfolio for this contract. [16 marks]

QUESTION B3

a.) Define an Exotic option contract. [4 marks]

b.) An American call option derivative expiring in 3-years has an exercise price of E1500.00 on the Eswatini stock market and currently trades at E1840.00. It is anticipated that the stock will rise by a factor of 1.05 and fell by a factor of 0.70. If the interest rate is 3%, Find the upward upward upward price of the option at the third year and its pay-off. [16 marks]

QUESTION B4.

(a.) Define a self financed derivative. [5 marks]

(b.) Evaluate the following open derivative claims:

i. $I = \int tB(t)dB(t)$ [5 marks]

ii. $I = \int B(t)dB(t)$ [5 marks]

iii. $I = \int B^3(t)dB(t)$ [5 marks]

QUESTION B5.

(a.) Define a derivative arbitrage. [5 marks]

(b.) A derivative market $X(t)$ is likely to contain arbitrage portfolio in trade. Given that

$$dX_0(t) = 0$$

$$dX_1(t) = 4dt + 3dB_1(t) + dB_2(t) - dB_3(t)$$

$$dX_2(t) = dt + dB_1(t) + 2dB_2(t) + 4dB_3(t)$$

$$dX_3(t) = dt + dB_1(t) + 2dB_2(t) + 5dB_3(t)$$

Assist the managers on whether or not there is pricing arbitrage. [15 marks]

QUESTION B6.

(a.) Find the price of a derivative whose changes follow the Black-Scholes equation

$dX(t) = \alpha X(t)dt + \rho X(t)dB(t); B(0) = 0, \alpha \in \Re.$ [15 *marks*]

(b.) Evaluate the Expected price if the expiration time of the derivative is at $T = 25$ years. [5 marks]

END OF EXAMINATION