UNIVERSITY OF SWAZILAND

MAIN L EXAMINATION, 2020/2021

MSc.I

Title of Paper : Special Topics In Financial Mathematics

Course Number : MAT622

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions.
2. Answer ANY FOUR (4) questions.
3. Show all your working.
4. Start each new major question on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

ANSWER ANY *FOUR* QUESTIONS

QUESTION A1 [*25 Marks*]

(a) A perpetuity consists of yearly increasing payments of $(1+jk), (1+j)2, (1+j)3$, etc ., commencing at the end of the first year . At an annual effective interest rate of 4%, the present value one year before the first payment is 51. Determine j,

[9 Marks]

(b) If $v(t) = 2^{-t}$, and given cash flow vectors $\mathbf{c} = (1, 2, 3)$ and $\mathbf{e} = (2, K, 1)$. Find K so that $\mathbf{c}$ and $\mathbf{e}$ are actuarially equivalent with respect to v.

[6 Marks]

(c) The death benefit on a life insurance policy can be paid in four ways. All payments have the same present value:

(i) A perpetuity of 120 at the end of each month, first payment one month after the moment of death;

(ii) Payments of 365.47 at the end of each month for n years, first payment one month after the moment of death;

(iii) A payment of 17,866.32 at the end of n years after the moment of death; and

(iv) A payment of X at the moment of death.

Calculate X.

[10 Marks]

QUESTION A2 [*25 Marks*]

(a) A survival model follows Makeham's law, so that

$$\mu_x = A + Bc^x \quad \text{for } x \geq 0.$$

(i) Find ${}_tp_x$

[5 Marks]

(ii) Suppose you are given the values of ${}_{10}p_{50}$, ${}_{10}p_{60}$ and ${}_{10}p_{70}$. Find c.

[6 Marks]

(b) A life insurer assumes that the force of mortality of smokers at all ages is twice the force of mortality of non-smokers.

(i) Show that, if * represents smokers' mortality, and the 'unstarred' function represents non-smokers' mortality, then

$$ {}_tp_x^* = ({}_tp_x)^2 $$

[7 Marks]

(ii) Calculate the variance of the future lifetime for a non-smoker aged 50 and for a smoker aged 50 under Gompertz' law

[7 Marks]

QUESTION A3 [*25 Marks*]

(a) Assume that the forces of mortality and interest are each constant and denoted by μ and δ, respectively. Determine $Var(v^T)$ in terms of μ and δ.

[7 Marks]

(b) Given:

(a) The survival function is $S(x) = 1 - x/100$ for $0 \leq x \leq 100$..

(b) The force of interest is $\delta = 0.10$.

(c) The death benefit is paid at the moment of death.

Calculate the net single premium for a 10−year endowment insurance of 50,000 for a person age $x = 50$.

[8 Marks]

1. A 3−year term life insurance to (x) is defined by the following table

Year t	Death Benefit	q_{x+t}
0	3	0.20
1	2	0.25
2	1	0.50

Given: $v = 0.9$, the death benefits are payable at the end of the year of death and the expected present value of the death benefit is Π. Calculate the probability that the present value of the benefit payment that is actually made will exceed Π.

[10 Marks]

QUESTIONA4 [*25 Marks*]

(a) A farmer annually produces x_k units of a certain crop and stores $(1 - u_k)x_k$ units of his production, where $0 \leq u_k \leq 1$, and invests the remaining $u_k x_k$ units, thus increasing the next years production to a level x_{k+1} given by

$$x_{k+1} = x_k + W_k u_k x_k, \quad k = 0.1, \cdots, N-1$$

The scalars W_k are bounded independent random variables with identical probability distributions that depend neither on x_k nor on u_k. Furthermore, $E\{W_k\} = \bar{w} > 0$. The problem is to find the optimal policy that maximizes the total expected product stored over N years:

$$E\left(X_N + \sum_{k=0}^{N-1}(1 - u_k)X_k\right).$$

(a) Compute $J(T-1, x_{T-1})$.

[5 Marks]

(b) Compute $J(T-2, x_{T-2})$.

[5 Marks]

(c) Show that an optimal control law is given by:

(i) If $\bar{w} > 1$, then $u_0(x_0) = \cdots = u_{N-1}(x_{N-1}) = 1$.

(ii) If $0 < \bar{w} < 1/N$, then $u_0(x_0) = \cdots = u_{N-1}(x_{N-1}) = 0$.

[15 Marks]

QUESTION A5 [*25 Marks*]

(a) Z is the present-value random variable for an insurance on the independent lives of (x) and (y) where

$$Z = \begin{cases} v^{T(y)}, & \text{if } T(y) \leq T(x) \\ 0, & \text{otherwise} \end{cases}$$

i. (x) is subject to a constant force of mortality of 0.07.
ii. (y) is subject to a constant force of mortality of 0.09.
iii. The force of interest is a constant $\delta = 0.06$.

Calculate $Var(Z)$.

[8 Marks]

(b) Given the joint distribution of claims Y_1 and Y_2. Compute the probability distribution of $Y_1 + Y_2$.

	Y_1	
Y_2	0	1
0	0.38	0.17
1	0.14	0.02
2	0.24	0.05

[9 Marks]

(c) A fully discrete last-survivor insurance of 1 is issued on two independent lives each age x . Level net annual premiums are paid until the first death. Given:

i. $A_x = 0.4$
ii. $A_{xx} = 0.55$
iii. $a_x = 9.0$

Calculate the net annual premium

[8 Marks]

QUESTION A6 [*25 Marks*]

(a) The aggregate claims S are approximately normally distributed with mean μ, and variance σ^2. Show that thep-loss reinsurance net premium $\rho(\beta) = E[(X - \beta)^+]$ is given by

$$\rho(\beta) = (\mu - \beta)\Phi\left(\frac{\mu - \beta}{\sigma}\right) + \sigma\phi\left(\frac{\mu - \beta}{\sigma}\right)$$

where Φ and ϕ are the standard normal distribution and density functions.

[10 Marks]

(b) Consider the compound model described by formula $S = X + \ldots + X_N$ where N, X_i are independent, and X_i are identically distributed. Show that $E[S] = E[N]E[X]$ and

$$E[S^2] = E[N^2]E[X]^2 + E[N](E[X^2 - E[X]^2).$$

where $M_N(t)$ where $M_x(t)$ are the moment generating functions of N and X.

[15 Marks]

END OF EXAMINATION PAPER