University of Eswatini

Main Examination, 2020/2021

M.Sc MATH

Title of Paper : Environmental Fluid Mechanics

Course Code : MAT631

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN questions answer **ANY FIVE questions.**
2. Each question worth 20 marks.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

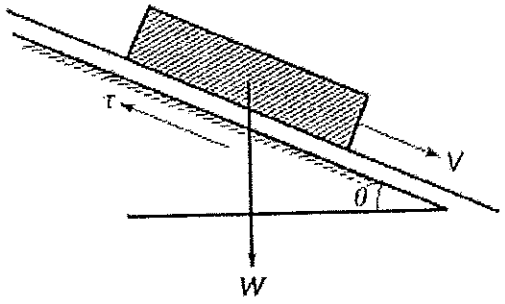
(a) Calculate the velocity component v if $u = Ae^x$ is the other component so that the equation of continuity satisfied. [5]

(b) A steady, incompressible, two dimensional velocity is given by

$$u = a^2 - (b - cx)^2, v = -2cby + 2c^2xy.$$

Determine the location of the stagnation point, if it exists. [5]

(c) A $90N$ rectangular solid block slides down a 30^0 inclined plane as shown in the figure below. The plane is lubricated by a $3mm$ thick film of oil of relative density of 0.90 and viscosity 8.0 *poise*. If the contact area is $0.3m^2$, estimate the terminal velocity of the block. [10]

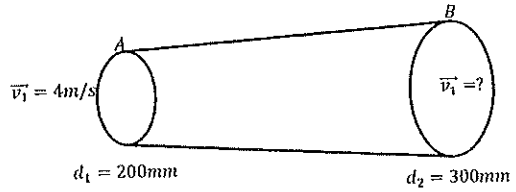


Question 2

(a) The diameters of a pipe at the sections A and B are 200 mm and 300 mm respectively see the diagram below. If the velocity of water flowing through the pipe at the section A is $4m/s$, find

(i) discharge through the pipe, [5]

(ii) velocity of water at section B. [5]



(b) The velocity distribution of a certain two-dimensional flow is given by $u = Ay + B$ and $v = Ct$, where A, B, C are constants. Obtain the equation of the motion of fluid particles using the Lagrangian method. [10]

Question 3

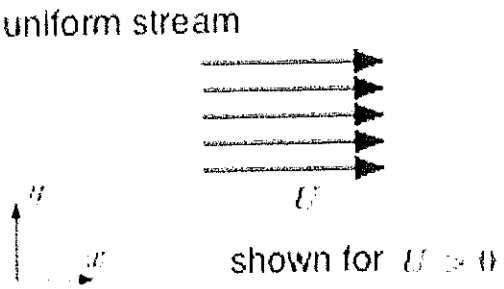
(a) The resistance force F of a ship is a function of its length L, velocity V, acceleration due to gravity g and fluid properties like density ρ and viscosity μ. Write this relationship in a dimensionless form.
(Hint: write $F = f(L, V, g, \rho, \mu)$) [10]

(b) Consider a uniform flow in positive x-direction as shown in the figure below. with $u = U$ and $v = 0$.

(i) Show that the flow is physically possible incompressible flow. [2]

(ii) Show that the flow is irrotational. [2]

(iii) Determine the stream and potential functions. [6]



Question 4

For a two dimensional flow, the velocity function is given by the expression, $\phi = x^2 - y^2$.

(i) Determine velocity components in x and y directions. [4]

(ii) Show that the velocity components satisfy the conditions of flow continuity and irrotationality. [5]

(iii) Determine stream functions and flow rate between the streamlines $(2, 0)$ and $(2, 2)$. [5]

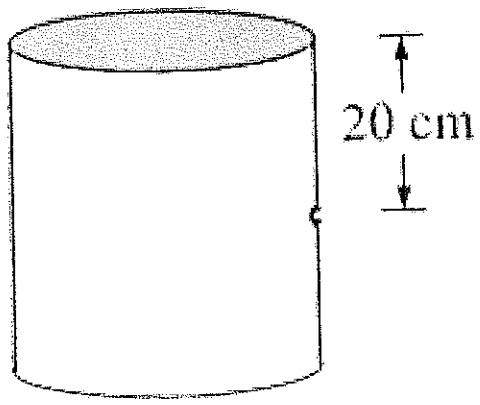
(iv) Show that the streamlines and potential lines intersect orthogonally at the point $(2, 2)$. [6]

Question 5

(a) What are the three major assumptions used in the derivation of the Bernoulli equation. [3]

(b) Write down Bernoulli's equation, what does each term represent? [5]

(c) A Styrofoam cylinder, filled with water, sits on a table. You then poke a small hole through the side of the cylinder, 20 cm below the top of the water surface as shown in the figure. What is the speed of the fluid emerging from the hole? [12]



Question 6

(a) Define mass and volume flow rates. [4]

(b) Name and briefly describe the four fundamental types of motion or deformation of fluid particles. [4]

(c) A steady, incompressible, two-dimensional velocity field is given by

$$V = (u, v) = (0.5 + 0.8x)\mathrm{i} + (1.5 - 0.8y)\mathrm{j}.$$

Calculate the various kinematics properties, namely, the rate of translation, rate of rotation, linear strain rate and shear strain rate. [12]

Question 7

(a) (i) What is the basic assumption for a creeping flow? [2]

(ii) Define Hele-Shaw flow. [2]

(iii) Write a one-word description of each of the five terms in the incompressible Navier-Stokes equation,

$$\underbrace{\rho\frac{\partial \vec{V}}{\partial t}}_{\text{I}} + \underbrace{\rho(\vec{V}\nabla)\vec{V}}_{\text{II}} = \underbrace{-\vec{\nabla}P}_{\text{III}} + \underbrace{\rho\vec{g}}_{\text{IV}} + \underbrace{\mu\nabla^2\vec{V}}_{\text{V}}$$

When the creeping flow approximation is made, only two of the five terms remain. Which two terms remain, and why is this significant? [8]

(b) Consider a viscous Newtonian fluid on top of an infinite flat plate lying in the xy-plane at $z = 0$. The fluid is at rest until time $t = 0$, when the plate suddenly starts moving at speed V in the x-direction. Gravity acts in z-direction. Formulate the one dimensional diffusion equation and determine the pressure field. [8]

End of examination paper