UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2005

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: **CHOOSE AND ATTEMPT ANY FOUR QUESTIONS. EACH QUESTION CARRIES 25 MARKS.**

THE PAPER CONSISTS OF EIGHT PAGES INCLUDING THIS ONE. DATA SHEET IS ALSO INCLUDED.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

Question 1

a) (i) If the current in an inductor is doubled by what factor does the stored energy change? (2 Marks)

(ii) Why is the induced emf that appear in an inductor called a 'counter' or a 'back' emf? (3 Marks)

b) An LC circuit has an inductance of 2.81 mH and a capacitance of 9 pF as shown in Fig 1.1. The capacitor is initially charged with a 12-V battery when the switch S_1 is open and switch S_2 is closed. S_1 is then closed at the same instant that S_2 is opened so that the capacitor is shorted across the inductor.

(i) Find the frequency of oscillation of current. (2 Marks)

(ii) What are the maximum values of charge on the capacitor and current in the circuit? (4 Marks)

(iii) Express the charge and current as functions of time. (4 Marks)

(iv) What is the total energy stored in the circuit? (4 Marks)

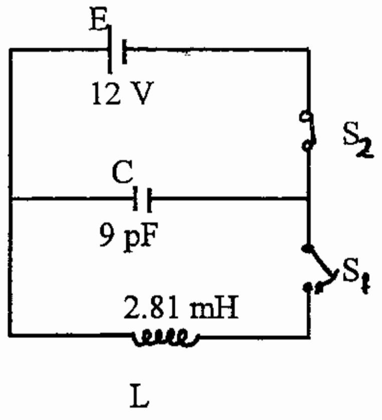

Fig 1.1

c) An LC circuit carries a current that is oscillating with a period T. If the charge on the capacitor is at a maximum at t=0, when will the energy stored in the electric field equal the energy stored in the magnetic field of the inductor? (Express your answer as a fraction of T.) (6 Marks)

Question 2

a) (i) If the frequency is doubled in a series RLC circuit, what happens to the resistance, the inductive reactance and the capacitive reactance? (6 Marks)

(ii) What is the impedance of an RLC circuit a the resonance frequency? (2 Marks)

b) The RC low-pass filter shown in Fig 2.1 has a resistance R=90 Ω and a capacitance C=8000 pF. Calculate the gain (V_{out}/V_{in}) and phase angle for an input frequency:

(i) f=600 Hz. (5 Marks)

(ii) f=600kHz . (5 Marks)

(iii) Comment on the two results above. (2 Marks)

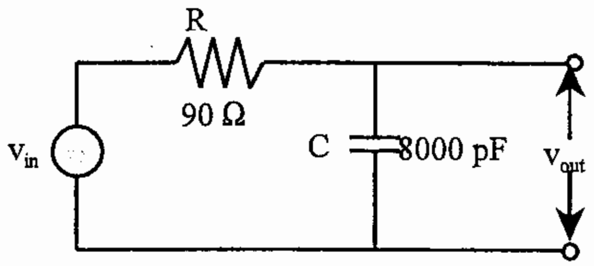

Fig 2.1

c) An RLC circuit consists of a 150-Ω resistor, a 21-μF capacitor and a 460-mH inductor, connected in series with a 120-V, 60 Hz power supply.

(i) What is the phase angle between the current and the applied voltage? (3 Marks)

(ii) Does the current or the voltage reach its peak earlier? (2 Marks)

Question 3

a) (i) Discuss how mutual inductance arises between the primary and the secondary coils in a transformer. (6 Marks)

(ii) Show that the inductive time constant τ has SI units of seconds. (3 Marks)

b) A 10-V battery, a 5-Ω resistor, and a 10-H inductor are connected in series. After the current in the circuit has reached its maximum value, calculate:

(i) the power supplied to the circuit by the battery. (3 Marks)

(ii) the power dissipated in the resistor. (3 Marks)

(iii) the power dissipated in the inductor. (2 Marks)

(iv) the energy stored in the magnetic field of the inductor. (4 Marks)

c) An emf of 96 mV is induced in the winding of a coil when the current in a nearby coil is increasing at the rate of 1.2 A/s. What is the mutual inductance of the coils?

(4 Marks)

Question 4

a) (i) A messy loop of wire is placed on a smooth table and anchored at points a and b as shown in Fig 4.1. If a current i is now passed through the wire, will it try to form a circular loop or will it try to bunch up further?

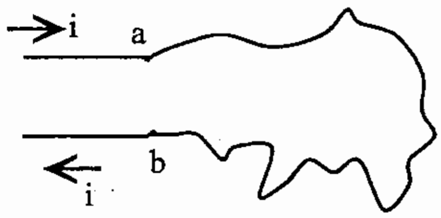

Fig 4.1

(2 Marks)

(ii) Consider two charges, first of the same sign and then of opposite signs, that are moving along separate parallel paths with the same velocity. Compare the directions of the mutual electric and magnetic forces in each case. (4 Marks)

b) Use the Biot-Savart law to calculate the magnetic field B at C, the common centre of the semicircular arcs AD and HJ in Fig 4.2. The two arcs of radius R2 and R1, respectively, form part of the circuit ADJHA carrying current i.

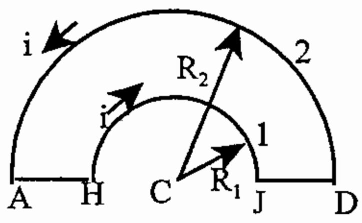

Fig 4.2

(7 Marks)

c) Fig 4.3 shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the resultant magnetic field at point P? Assume the following values: i_1=15 A, i_2=32 A, and d=5.3 cm.

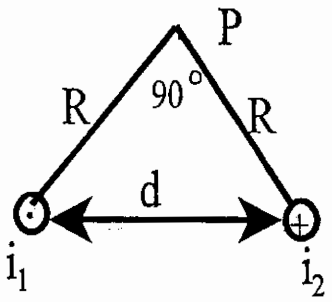

Fig 4.3

(12 Marks)

Question 5

a) (i) When is it necessary to approximate a charge distribution by a point charge? (2 Marks)

(ii) A 'free' electron and 'free' proton are placed in an identical electric field. Compare the electric forces on each particle. Compare their acceleration. (4 Marks)

(b) Two small silver spheres, each with a mass of 100 g are separated by a distance of 1 m. Calculate the fraction of the electrons in one sphere that must be transferred to the other in order to produce an attractive force of 10^4 N between the spheres. (The number of electrons for an atom of silver is 47 and the number of atoms per gram is Avogadro's number divided by the atomic weight of silver, 107.87). (10 Marks)

(c) Assume an isolated point charge q. Starting with Gauss' law, calculate the electric field due to that change. Show that Coulomb's law follows from this result. (9 Marks)

Question 6

a) (i) Two different sets of Xmas-tree lights are available. For set A, when one bulb is removed (or burns out) the remain bulbs remain illuminated. For set B, when one bulb is removed, the remaining bulbs fail to operate. By means of diagram(s) explain the difference in the wiring of the two sets.

(4 Marks)

(ii) Embodied in Kirchhoff's are two conservation laws. What are they? Explain.

(4 Marks)

a) Calculate the potential difference between the points a and b for the circuit shown in Fig 6.1 and identify which point is at the higher potential. (8 Marks)

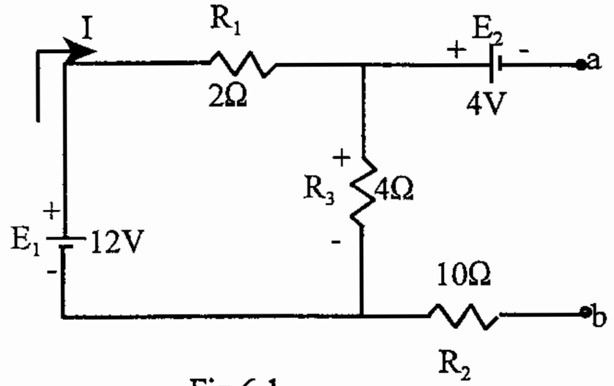

Fig 6.1

b) (i) Describe how, by means of a well-labelled diagram, a galvanometer with resistance R_G can be modified so that it can be used to measure current. What are the relative values of the necessary circuit elements. (4 Marks)

(ii) Assume that a galvanometer has an internal resistance of 60 Ω and requires a current of 0.5 mA to produce a full-scale deflection. What value of resistance must be connected in parallel with the galvanometer if the combination is to serve as an ammeter with a full-scale deflection for a current of 0.1 A. Use Kirchhoff's laws. (5 Marks)

Some Fundamental Constants[a]

Quantity	Symbol	Value[b]
Atomic mass unit	u	$1.660\ 540\ 2(10) \times 10^{-27}$ kg $931.434\ 32(28)$ MeV/c^2
Avogadro's number	N_A	$6.022\ 136\ 7(36) \times 10^{23}$ (g mol)$^{-1}$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.274\ 015\ 4(31) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k}$	$0.529\ 177\ 249(24) \times 10^{-10}$ m
Boltzmann's constant	$k = R/N_A$	$1.380\ 658(12) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$2.426\ 310\ 58(22) \times 10^{-12}$ m
Deuteron mass	m_d	$3.343\ 586\ 0(20) \times 10^{-27}$ kg $2.013\ 553\ 214(24)$ u
Electron mass	m_e	$9.109\ 389\ 7(54) \times 10^{-31}$ kg $5.485\ 799\ 03(13) \times 10^{-4}$ u $0.510\ 999\ 06(15)$ MeV/c^2
Electron-volt	eV	$1.602\ 177\ 33(49) \times 10^{-19}$ J
Elementary charge	e	$1.602\ 177\ 33(49) \times 10^{-19}$ C
Gas constant	R	$8.314\ 510(70)$ J/K·mol
Gravitational constant	G	$6.672\ 59(85) \times 10^{-11}$ N·m^2/kg^2
Hydrogen ground state	$E_0 = \frac{m_e e^4 k^2}{2\hbar^2} = \frac{e^2 k}{2a_0}$	$13.605\ 698(40)$ eV
Josephson frequency-voltage ratio	$2e/h$	$4.835\ 976\ 7(14) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \frac{h}{2e}$	$2.067\ 834\ 61(61) \times 10^{-15}$ Wb
Neutron mass	m_n	$1.674\ 928\ 6(10) \times 10^{-27}$ kg $1.008\ 664\ 904(14)$ u $939.565\ 63(28)$ MeV/c^2
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	$5.050\ 786\ 6(17) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2 (exact)
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 817 \times 10^{-12}$ C^2/N·m^2 (exact)
Planck's constant	h $\hbar = h/2\pi$	$6.626\ 075(40) \times 10^{-34}$ J·s $1.054\ 572\ 66(63) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\ 623(10) \times 10^{-27}$ kg $1.007\ 276\ 470(12)$ u $938.272\ 3(28)$ MeV/c^2
Quantized Hall resistance	h/e^2	$25812.805\ 6(12)$ Ω
Rydberg constant	R_H	$1.097\ 373\ 153\ 4(13) \times 10^7$ m^{-1}
Speed of light in vacuum	c	$2.997\ 924\ 58 \times 10^8$ m/s (exact)

[a] These constants are the values recommended in 1986 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see Cohen, E. Richard, and Barry N. Taylor, *Rev. Mod. Phys.* 59:1121, 1987.
[b] The numbers in parentheses for the values below represent the uncertainties in the last two digits.