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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2005

TITLE O F PAPER: MODERN PHYSICS & WAVE OPTICS

COURSE NUMBER: P231

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) The classical model of blackbody radiation given by the Rayleigh-Jeans law has two major flaws. Identify them and explain how Planck's law deals with them. **(4 marks)**

(b) A star moving away from the earth at 0.280*c* (*c* = speed of light in vacuum) emits radiation that we measure to be most intense at the wavelength 500 nm. Determine the surface temperature of this star. **(6 marks)**

(c) Why does the existence of the cutoff frequency in the photoelectric effect favour a particle theory for light rather than the wave theory? **(3 marks)**

(d) X- rays of wavelength λ = 0.0700 nm are Compton scattered by electrons in a graphite target at an angle of 120° with their original direction.

(i) What is the wavelength of the scattered electrons? **(3 marks)**
(ii) What is the kinetic energy in eV of the scattered electrons if they were initially at rest? **(2 marks)**
(iii) What is the scattering angle of the electrons? **(7 marks)**

QUESTION 2

(a) A mass m = 2 kg is attached to a spring of negligible mass that has a force constant k = 25 N/m. The spring is stretched 0.4 m from equilibrium and released.

(i) Find the quantum number for the system, assuming that it is quantized. **(6 marks)**

(ii) How much energy is emitted in one quantum change? **(2 marks)**

(b) Can an electron in the ground state of the hydrogen atom absorb a photon of energy less than 13.6 eV or greater than 13.6 eV? State the conditions under which the absorption can occur. **(5 marks)**

(c) What do you understand by a "hydrogen-like atom" and how does it differ from hydrogen? **(3 marks)**

(d) This problem is based on a single electron beryllium ion (Be^{3+}), Z = 4.

(i) Develop an equation for the energy levels of Be^{3+}. **(5 marks)**

(ii) What is the ionisation energy for Be^{3+}? **(2 marks)**

(iii) What is the radius of the first Bohr orbit for Be^{3+}? **(2 marks)**

QUESTION 3

(a) Starting with the equation for the decay rate of *N* nuclei

$$\frac{dN}{dt} = -\lambda N,$$

where λ is the decay constant, show that the half-life of a radioactive substance is given by

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$ **(6 marks)**

(b) A radioactive sample contains 3.50 μg of pure carbon-11 $\left({}^{11}_{6}C\right)$, which has a half-life of 20.4 min.

(i) What is the number of nuclei in the sample? **(3 marks)**
(ii) Find the initial activity of the sample in becquerels. **(2 marks)**
(iii) Find the activity after exactly eight hours. **(2 marks)**
(iv) What is the number of remaining ${}^{11}_{6}C$ nuclei after eight hours. **(2 marks)**

(c) Consider a neutron with an energy above 1 Mev travelling through matter. Discuss how such a neutron can be captured and write down the equation for neutron capture and explain it. **(5 marks)**

(d) Sketch a potential energy function as a function of separation for two deuterons and explain it. **(5 marks)**

QUESTION 4

(a) (i) Two wavelengths λ and $\lambda + \Delta\lambda$ ($\Delta\lambda << \lambda$) are incident on a diffraction grating of slit spacing *d*. Show that the angular separation between the m^{th} order spectra is given by

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{(d/m)^2 - \lambda^2}}.$$ **(6 marks)**

(ii) Sodium yellow light consist of the two wavelengths 589.0 nm and 589.6 nm. This light falls normally on a plane diffraction grating with 1500 rulings per cm. What is the angular separation of the two lines observed in the first order spectrum. **(4 marks)**

(b) Discuss with the aid of diagrams the process of polarization by double refraction. **(12 marks)**

(c) Explain what a half-wave plate is. **(3 marks)**

QUESTION 5

(a) A helium-neon laser with an optical cavity exactly 25 cm long is vibrating in the TEM_{00} mode with a wavelength of 632.8 nm.

(i) What is the number of loops in the standing wave pattern? **(3 marks)**
(iii) Find the frequency difference between modes. **(3 marks)**

(b) (i) Show that

$$\frac{\Delta \nu}{\nu} = -\frac{\Delta \lambda}{\lambda},$$

where ν is frequency and λ is wavelength. **(3 marks)**

(ii) The sodium line at 589.0 nm produced in a low pressure discharge, has a Doppler width of 0.00194 nm. Calculate the frequency of the light. Find the line width in hertz. **(3 marks)**

(iii) Calculate the coherence length. **(2 marks)**

(c) The solar constant, the power due to radiation from the sun falling on the earth's atmosphere, is 1.35 kW/m^2. Determine the magnitudes of the electric field E and the magnetic induction B of sunlight. **(8 marks)**

(d) A point source produces 20 watts of radiation 3 m above a table. Find the illuminance of the table directly below the light source. **(3 marks)**

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

σ = 5.669 6 x 10^{-8} W/(m^2K^2)^3
Boltzmann's constant, k_B = 1.3801 x 10^{-23} J/K
Bohr magneton, μ_B = 9.27 x 10-24 J/T
Speed of light in vacuum, c = 2.997 924 58 x 10^8 m/s
Planck's constant, h = 6.626 075 x 10^{-34} Js
$\hbar$ = 1.054 572 x 10^{-34} Js
hc = 1.986 447 x 10^{-25} Jm
$2\pi hc^2$ = 3.741 859 x 10^{-15} Jm/s
mass of an electron, m_e = 9.109 389 7 x 10^{-31} kg
mass of a proton, m_p = 1.672 623 x 10^{-27} kg
mass of a neutron, m_n = 1.674 928 6 x 10^{-27} kg
Coulomb constat, k_e = 8.987 551 787 x 10^9 Nm^2/C^2
electron charge, e = 1.602 177 33 x 10^{-19} C
1 atomic mass unit = 1 amu = 1 u = 1.660 540 2 x 10^{-27} kg = 931.494 MeV
1 eV = 1.602 177 33 x 10^{-19} J
$T_{1/2}(^{14}C)$ = 5730 years

$$\frac{N(^{14}C)}{N(^{12}C)} = 1.3x10^{-12}$$

Helium (^4He) mass = 4.002 602 u
Radon (^{222}Rn) mass = 222.017 571 u
Radium (^{226}Ra) mass = 226.025 402 u

$$\lambda_{\max} = \frac{hc}{4.965kT}$$

$\theta_{min} = 1.22\lambda/D$

$$I = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kt}} - 1\right)}$$

$$\int \cos^2 u du = \frac{u}{2} + \frac{1}{4}\sin 2u$$