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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2005

TITLE OF THE PAPER: CLASSICAL MECHANICS

COURSE NUMBER : P320

TIME ALLOWED : THREE HOURS

INSTRUCTIONS:

ANSWER ANY FOUR OUT OF FIVE QUESTIONS.

EACH QUESTION CARRIES 25 MARKS.
MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 4 PAGES, INCLUDING THIS PAGE.

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Q.1. The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + f y^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2+y^2}$$

where a, b, c, f, g and k are constants.

(i) List the generalized coordinates. [2]

(ii) Write down the equations of motion. [10]

(iii) What is the Hamiltonian? [10]

(iv) What quantities are conserved? [3]

Q.2. (i) A particle of mass m is under the influence of potential [7]

$V(r) = -\frac{k}{r}$. Show that the total energy $E = \frac{m\dot{r}^2}{2} + \frac{\ell^2}{2mr^2} - \frac{k}{r}$.

(ii) If the launching velocity of a satellite from the surface of the earth is $v < v_e$ where v_e is the escape velocity, show that the maximum height h that can be obtained for vertical launch is given by [10]

$$h = \frac{2R^2 g}{2Rg - v^2}$$

where *R* is the radius of the earth.

(iii) Show that for a geo-stationary satellite of mass *m* and at distance *r* from the centre of earth is given by [8]

$$r^3 = \frac{GMT^2}{4\pi^2}$$

where G= gravitational interaction constant, M= mass of earth and T is the period of revolution.

Q.3. (i) Bounded orbits are possible only for attractive potential $V(r) = -\frac{k}{r}$ and orbits are ellipse or circle with two turning points r_1 and r_2.

Semi-major axis $a = \frac{r_1 + r_2}{2}$ and semi minor axis $b = a\sqrt{1-\varepsilon^2}$ where

$\varepsilon = \sqrt{1 + \frac{2E\ell^2}{mk^2}}$ where ℓ is orbital angular momentum.

Show that $\tau^2 = \frac{4\pi^2 m}{k}a^3$ where τ is the period of the orbit and *m* is the reduced mass. [15]

(ii) The major axis of the elliptic orbit of a certain comet is 100 astronomical units.

(a) What is its period? [3]

(b) If its distance from the sun is ½ astronomical unit at perihelion, what is the value of the eccentricity of the orbit? [5]

(c) What is the comets's speed at perihelion and aphelion? [2]

***Note:** Assume that mass of sun is very large compared to the mass of comet.*

Q.4. (A) Consider free small oscillations in one dimension about a position of stable equilibrium of a particle with mass m and displacement as x .

(i) Show that the potential is given by $\frac{1}{2}kx^2$. [5]

(ii) Derive an expression for the Lagrangian. [2]

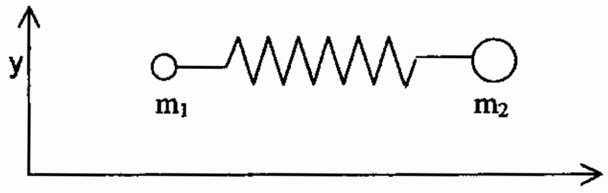
(iii) Derive the equations of motion for this system.

(iv) Show that angular frequency depends only on the property of the mechanical system. [2]

(v) Show that the total energy $E = \frac{1}{2} m \omega^2 a^2$, where $\omega = \sqrt{\frac{k}{m}}$ [3]

and a is the amplitude.

(B) A typical diatomic molecule may be regarded as equivalent to two masses m_1 and m_2 connected by a massless spring of spring constant k and of unstretched length a , vibrating along the line joining the two masses as shown



Taking coordinates of m_1 and m_2 as x_1 and x_2 respectively,

(i) write down the expressions for potential and kinetic energy. [2]

(ii) write down the equation of motion. [3]

(iii) show that the system has two normal angular velocities [8]

$$\omega_1 = 0 \quad \text{and} \quad \omega_2 = \pm\left(\frac{k(m_1 + m_2)}{m_1 m_2}\right)^{1/2}$$

Q.5. (A) To an observer in the rotating system, rotating with angular velocity ω , a moving particle is under the influence of an effective force

$$\vec{F}_{eff} = \vec{F} - 2m(\vec{\omega} \times \vec{V}_r) - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

where $\vec{F}$ = force in the inertial system,

$\vec{V}_r$ = velocity of the particle in the rotating set of axes.

(i)Which is the term in the above expression corresponding to Coriolis force? [1]

(ii)What is the physical interpretation of the last term? [1]

(iii)Earth rotates with $\omega = 7.292 \times 10^{-5}$ *rad/s*. Calculate the value of centripetal acceleration at the equator where $r = 6.378 \times 10^{6}$ *m.*

Explain the effect of this acceleration on a falling body. [3]

(iv)At the Northern Hemisphere, what is the direction of ω. [5]
Fixing the z-axis along this direction, diagrammatically illustrate the deflection of a projectile shot along the earth's surface.
(v)What will be the direction of the projectile if shot along the earth's surface at Sothern Hemisphere and at the Equator? [2]

(B) For a force free motion of a symmetrical top, Euler equations are

$$I_1 \dot{\omega}_1 = (I_1 - I_3)\omega_3 \omega_2$$

$$I_2 \dot{\omega}_2 = -(I_1 - I_3)\omega_3 \omega_1$$

$$I_3 \dot{\omega}_3 = 0$$

and $\omega^2 = \omega_1^2 + \omega_2^2 + \omega_3^2$.

(i) Show that $\omega_1 = A\cos(\Omega t)$ and $\omega_2 = A\sin(\Omega t)$ [7]
Where $\Omega = \frac{(I_3 - I_1)}{I_1}\omega_3$ and A is a constant.

(ii) Show that magnitude of $\vec{\omega}$ is constant. [2]

(iii) The earth can be approximated as symmetrical about the polar axis and slightly flattened at the poles so that $I_1 < I_3$.
Numerically, $\frac{I_3 - I_1}{I_1} \approx 0.00327$.
Assuming $\omega_3 \approx \omega$, calculate the period predicted for the precision of the axis of rotation [4]