201

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2005

TITLE OF THE PAPER: COMPUTATIONAL PHYSICS-II

COURSE NUMBER : P482

TIME ALLOWED : *SECTION A: ONE HOUR.*
SECTION B: TWO HOURS.

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS PAPER:

***SECTION A* : THIS IS WRITTEN PART ON YOUR ANSWER BOOK.**
CARRIES A TOTAL OF 30 MARKS.
***SECTION B*: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT.**
CARRIES A TOTAL OF 70 MARKS.

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **BOTH** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS FOUR PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section)

Q.1.

(a) Explain the term random numbers. Why are the random numbers generated on a computer are called as psuedo-random numbers? [5]

(b) What do you understand by random walk ? Give a couple of practical examples where it can be used. [5]

(c) Explain the Monte-Carlo method of integration. What are its advantages ? [5]

Q.2.

(a) Define autocorrelation and show that the autocorrelation of a signal S with noise n will be a signal on a constant background due to noise. [10]

(b) A unit step function is defined as

$$f(t) = -1 \quad t < 0$$
$$= +1 \quad t > 0$$

Show that $f(t) = \frac{a_0}{2} + \frac{4}{\pi}\sum_{m=0}^{\infty}\frac{1}{(2m+1)}\sin[(2m+1)t]$. [5]

where a_0 is a constant.

Note: You have to use Fourier's series expansion.

Q.3. A liquid of low viscosity, such as water, flows from an inverted conical tank with circular orifice at the rate

$$\frac{dx}{dt} = -0.6\pi r^2\sqrt{-2g}\,\frac{\sqrt{x}}{A(x)}$$

where r is the radius of the orifice, x is the height of the liquid level from the vertex of the cone, and A(x) is the area of the cross section of the tank x units above the orifice. Initial water level of the tank is 2m .

Given: r =0.05 m, g=-9.8 ms^{-2} and A(x)=0.14 π x^2 .

Write a psuedo code to compute the water level after 10 minutes using RK-4^{th} order method with h=20s. [15]

SECTION B
(Practical Section)

Q.4. Consider a two dimensional lattice with 200 lattice points along the x-direction and 200 lattice points along the y-direction forming a set of square lattices. [35]
The distance between lattice points is of unit length. The total number of lattice points is 200 x 200 . Assume that all the lattice points at (x ,y=20) and (x ,y=-20) for all x have reflecting property. That is if a walker reaches y=20 the next step is towards south or if the walker reaches y=-20, the next step is towards north. Here we have assumed y-direction to be north-south.

Write a program and execute it to find the displacement of a walker taking 200 steps at random starting from (x=0,y=0).

Use the uniform random number generator available in Maple.

Q.5. The Duffing oscillator differential equation is given by

$$\frac{d^2 x}{dt^2} + k\frac{dx}{dt} + x^3 = B\cos t \qquad \text{where } B = \text{constant} = 5$$

Assume initial conditions to be at $t=0$, $x(t=0)=0.5$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.

(i) Write Maple commands to solve this equation numerically. [10]

(ii) Write an algorithm in pseudo code to find the solution x(t) on the interval $0 \le t \le 25$ at 200 points with a precision of 0.01.
You may use any one of the algorithms given in the Appendix.
Convert your algorithm into a program and execute it. [20]

(iii) Plot the solutions of (i) and (ii) on the same graph. [5]

@@@@END OF EXAMINATION@@@@

Appendix:

1. Solution of First Order Differential Equation with initial Conditions:

The equation is of the form $\frac{dy}{dx} = f(x,y)$ with the given initial boundary condition $y(x_0) = \alpha$.

(i) Euler's Method:

$$y_{i+1} = y_i + h\, f(x_i, y_i) \qquad \text{where } h = x_{i+1} - x_i$$

(ii) Fourth Order RK-Method:

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$k_1 = h\, f(x_i, y_i)$

$k_2 = h\, f\,(\,x_i + 0.5\,h\,,\,y_i + 0.5\,k_1\,)$
$k_3 = h\, f\,(\,x_i + 0.5\,h\,,\,y_i + 0.5\,k_2\,)$
$k_4 = h\, f\,(\,x_i + h\,,\,y_i + k_3\,)$ *and* $h = x_{i+1} - x_i$

2. Useful Integrals in Fourier Series and Fourier Transforms:

(A) Any arbitrary function f(t) can be expressed in terms of Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + \sum_{n=1}^{\infty} b_n \sin(nt)$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\,dt$$

For integer values of m and n,

$$\int_{-\pi}^{\pi} \sin(mt)\sin(nt)dt = \pi\,\delta_{mn} \qquad m \neq 0$$
$$= 0 \qquad m = n = 0$$

$$\int_{-\pi}^{\pi} \cos(mt)\cos(nt)dt = \pi\,\delta_{mn} \qquad m \neq 0$$
$$= 2\pi \qquad m = n = 0$$

$$\int_{-\pi}^{\pi} \sin(mt)\cos(nt)dt = \pi\,\delta_{mn} \qquad \text{all } m \text{ and } n.$$

(B) For integer values of m, n, and N,

$$\sum_{m=0}^{N-1} \exp(i\,2\pi mn/N) = N \qquad n = 0 \quad , \; i = \sqrt{-1}$$
$$= 0 \qquad n \neq 0$$

$$\sum_{m=0}^{N-1} \exp(i\,2\pi mn/N)\exp(-i\,2\pi mk/N) = N\,\delta_{kn}$$