UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: Answer four Questions. Each Question carries 25 Marks. Marks for each section are shown

Do not open this Paper until you told to do so by the **Invigilator**.

This Paper contains **EIGHT PAGES** including this one.

QUESTION 1

(a) Calculate the power dissipated in each resistor in the circuit of Fig. 1.1. (21 marks)

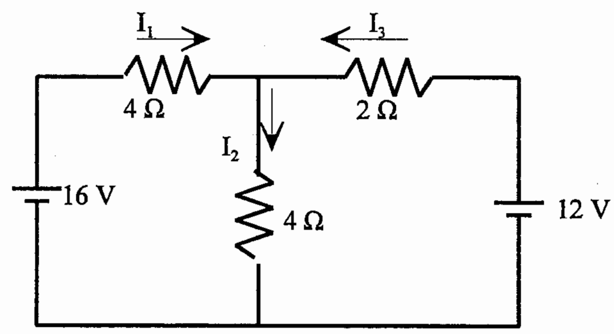

Fig 1.1

(b) Consider Fig. 1.2 in which a 1 - MΩ resistor, a 50 - μF capacitor and 30 - V voltage source are connected in series. Find

(i) the time constant of the circuit (2 marks)

(ii) the maximum charge on the capacitor after the switch is closed. (2 marks)

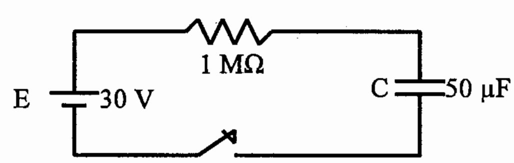

Fig 1.2

QUESTION 2

(a) A proton moves with a speed of 8×10^6 m/s along the x-axis. It enters a region where there is a magnetic field of magnitude 2.5 T directed at an angle of 60° to the x-axis and lying in the x-y plane.

(i) Draw the diagram of motion and in it show the direction of the force acting on the proton. (4 marks)

(ii) Calculate the initial magnetic force. (4 marks)

(iii) What is the acceleration of the proton? (4 marks)

(iv) Show that the units of the magnetic force reduce to newtons. (3 marks)

[The mass of proton is 1.67×10^{-27} kg.]

(b) (i) A proton is moving in a circular orbit of radius 14 cm in a uniform magnetic field of magnitude 0.35 T directed perpendicular to the velocity of the proton. Find the orbital speed of the proton. (5 marks)

(ii) What will be the radius of the circular orbit if the electron replaced the proton? Mass of an electron is 9.10×10^{-31} kg. (5 marks)

QUESTION 3

(a) A certain charged, parallel plate capacitor has vacuum between its plates. While the capacitor remains isolated so that the charge is unaffected, the plate separation is halved. By what factor does each of the following quantities charge as a result?

(i)	capacitance	(2 Marks)
(ii)	electric field	(2 Marks)
(iii)	stored energy	(2 Marks)
(iv)	potential difference	(2 Marks)

(b) A parallel-plate capacitor with air between the plates has a capacitance of 10 μF. If a dielectric material (K = 4) is inserted between the plates filling half the spacing between the plates, find the new capacitance. (11 Marks)

(c) (i) Coal has an energy density of $1.20 \times 10^{10}\ \mathrm{J/m^3}$. How large an electric field would be required to make an equal energy density in some region of space? (3 Marks)

(ii) In particular, if one wanted to create such an energy density between the plates of a parallel-plate capacitor whose plates are separated by 0.30 m, each plate having an area of 0.70 m^2, how big a potential difference would one need to set up between the plates? (3 Marks)

QUESTION 4

A certain long solenoid S has 220 turns/cm and carries a current given by

$$i = 3.0t + 1.2t^2$$

where i is in amperes and t in seconds. The diameter of the solenoid is 3.2 cm. At the centre, inside, of the solenoid is placed a 130-turn closely-packed coil C of diameter 2.1 cm.

(a) What flux crosses the coil? (8 marks)

(b) Plot the induced emf in the coil as function of time t, using the graph paper provided. (12 marks)

(c) The resistance of the coil is 0.15 Ω. What is the current in the coil at t = 0.2 s? (5 marks)

Question 5

(a) (i) If there are more electric field lines leaving a gaussian surface than there are entering the surface, what can you conclude about the net charge enclosed by that surface? (2 Marks)

(ii) If the charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss' law to find the electric field? Explain. (3 Marks)

(iii) A uniform electric field exists in a region of space in which there are no charges. what can you conclude about the net electric flux through a gaussian surface placed in this region of space? Explain. (2 Marks)

(b) A uniform electric field $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ intersects a surface of area A. What is the flux through this area if the surface lies

(i) in the yz plane? (3 Marks)

(ii) in the xz plane? (2 Marks)

(iii) in the xy plane? (2 Marks)

(c) A hollow rubber ball of inner radius a and outer radius b carries a charge uniformly distributed through the volume of the rubber, with charge per unit volume ρ. Find the electric field at a distance r from the centre of the ball, where

(i) $r < a$, (2 Marks)

(ii) $a < r < b$ (3 Marks)

(iii) $r > b$ (3 Marks)

(iv) For $r > b$, is the field the same as if all the charge were concentrated at the centre. Justify the answer mathematically. (3 Marks)

Question 6

a) Find the reactance of a 1.0-μF capacitor at a frequency of $200/\pi$ Hz and at $2\times10^6/\pi$ Hz. Make conclusions, if any, about the behaviour of a capacitor as a function of frequency in an AC circuit. (6 Marks)

b) Consider the circuit shown in Fig 6.1.

(i) Find the voltages across the various elements in the circuit in Fig 6.1. Assume the following values: R=1000Ω, L=2.0 H, and C=1μF. Let $v = \left(100\sqrt{2}\right)\sin 1000t$ (9 Marks)

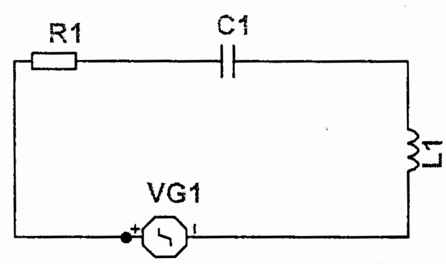

Fig 6.1

(ii) Could Kirchhoff's voltage law be verified in this case? Explain. (3 Marks)

c) Now let the circuit in Fig 6.1 have the following values: R=20Ω, C=1μF while L is unknown. The operating frequency is $360/2\pi$ Hz.

(i) How large an inductor must be added in series to make the phase angle for the circuit zero? (4 Marks)

(ii) What will the current in the circuit be if the applied voltage is 120V? (3 Marks)

Some Fundamental Constants[a]

Quantity	Symbol	Value[b]
Atomic mass unit	u	$1.660\ 540\ 2(10) \times 10^{-27}$ kg $931.434\ 32(28)$ MeV/c^2
Avogadro's number	N_A	$6.022\ 136\ 7(36) \times 10^{23}$ (g mol)$^{-1}$
Bohr magneton	$\mu_B = \dfrac{e\hbar}{2m_e}$	$9.274\ 015\ 4(31) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \dfrac{\hbar^2}{m_e e^2 k}$	$0.529\ 177\ 249(24) \times 10^{-10}$ m
Boltzmann's constant	$k = R/N_A$	$1.380\ 658(12) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \dfrac{h}{m_e c}$	$2.426\ 310\ 58(22) \times 10^{-12}$ m
Deuteron mass	m_d	$3.343\ 586\ 0(20) \times 10^{-27}$ kg $2.013\ 553\ 214(24)$ u
Electron mass	m_e	$9.109\ 389\ 7(54) \times 10^{-31}$ kg $5.485\ 799\ 03(13) \times 10^{-4}$ u $0.510\ 999\ 06(15)$ MeV/c^2
Electron-volt	eV	$1.602\ 177\ 33(49) \times 10^{-19}$ J
Elementary charge	e	$1.602\ 177\ 33(49) \times 10^{-19}$ C
Gas constant	R	$8.314\ 510(70)$ J/K·mol
Gravitational constant	G	$6.672\ 59(85) \times 10^{-11}$ N·m^2/kg^2
Hydrogen ground state	$E_0 = \dfrac{m_e e^4 k^2}{2\hbar^2} = \dfrac{e^2 k}{2a_0}$	$13.605\ 698(40)$ eV
Josephson frequency-voltage ratio	$2e/h$	$4.835\ 976\ 7(14) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \dfrac{h}{2e}$	$2.067\ 834\ 61(61) \times 10^{-15}$ Wb
Neutron mass	m_n	$1.674\ 928\ 6(10) \times 10^{-27}$ kg $1.008\ 664\ 904(14)$ u $939.565\ 63(28)$ MeV/c^2
Nuclear magneton	$\mu_n = \dfrac{e\hbar}{2m_p}$	$5.050\ 786\ 6(17) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2 (exact)
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 817 \times 10^{-12}$ C^2/N·m^2 (exact)
Planck's constant	h $\hbar = h/2\pi$	$6.626\ 075(40) \times 10^{-34}$ J·s $1.054\ 572\ 66(63) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\ 623(10) \times 10^{-27}$ kg $1.007\ 276\ 470(12)$ u $938.272\ 3(28)$ MeV/c^2
Quantized Hall resistance	h/e^2	$25812.805\ 6(12)\ \Omega$
Rydberg constant	R_H	$1.097\ 373\ 153\ 4(13) \times 10^7$ m^{-1}
Speed of light in vacuum	c	$2.997\ 924\ 58 \times 10^8$ m/s (exact)

[a] These constants are the values recommended in 1986 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see Cohen, E. Richard, and Barry N. Taylor, *Rev. Mod. Phys.* 59:1121, 1987.

[b] The numbers in parentheses for the values below represent the uncertainties in the last two digits.