UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2006

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: Answer four Questions. Each Question carries 25 Marks. Marks for each section are shown

Do not open this Paper until you told to do so by the **Invigilator**.

This Paper contains **EIGHT PAGES** including this one.

Question 1

a) (i) Explain why, under static conditions, all points in a conductor must be at the same electrostatic potential. (2 Marks)

(ii) The electric field inside a hollow uniformly charged sphere is zero. Does this mean that the potential is zero inside the sphere? Explain. (2 Marks)

b) A charge of +6 μC is placed at the point (3,0) and another charge of -2μC is placed at (-3,0).

(i) If the zero point of potential is taken at infinity, what is the potential of point (0,4)? (4 Marks)

(ii) Under these conditions, what is the potential of point (-5,0)? (4 Marks)

(iii) If, instead of taking zero of potential at infinity, you were to take it at the point (-5,0), what then would be the potential of the point (0,4)? (3 Marks)

(iv) Given the above charge setup, how much work does it take to move a +8μC charge from the point (0,4) to the point (0,0). (3 Marks)

Assume all distances are in metres.

c) Consider an electric field in a region of space given by

$$\vec{E} = (E_0 e^{3x})\hat{\imath},$$

where $E_0 = 54.3$ V/m. Determine the potential difference, $\Delta V = V_2 - V_1$ for the positions $r_1 = 4.3\,\mathbf{i}$ m and $r_2 = 3.4\,\mathbf{j}$ m. (7 Marks)

Question 2

a) (i) A pair of capacitors are connected in parallel while an identical pair are connected in series. Which pair would be more dangerous to handle after being connected to the same voltage source? Explain with simple derivations. (5 Marks)

(ii) What is the difference between a dielectric strength and the dielectric constant? (3 Marks)

b) For the circuit shown in Fig 2.1, do the following:

(i) Find the equivalent capacitance between points a and b. (3 Marks)

(ii) Find the potential difference across the 5-μF capacitor. (3 Marks)

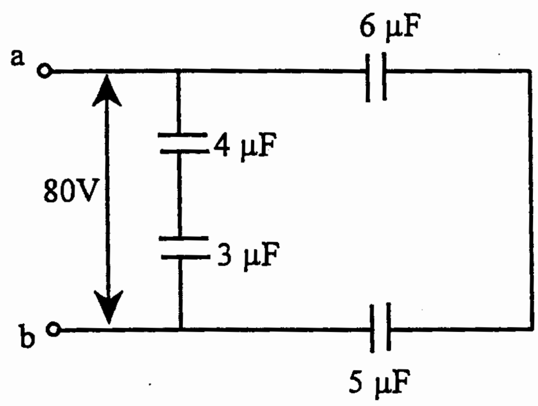

Fig 2.1

c) A parallel plate capacitor has plates of area A and separation d. The capacitor is connected to a battery of emf V. The plates are pulled apart until their separation is 2d, with the battery remaining connected throughout the process. Derive or state expressions for the following quantities:

(i) The initial charge q_i on the capacitor. (2 Marks)

(ii) The final charge q_f on the capacitor. (2 Marks)

(iii) The initial electrostatic energy U_{i0} stored in the capacitor. (2 Marks)

(iv) The final electrostatic energy U_{f0} store in the capacitor. (2 Marks)

(v) The change in the potential energy ΔU_b of the battery and whether this is an increase or a decrease. (3 Marks)

Question 3

a) (i) Two wires A and B of circular cross section are made of the same metal and are of equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross sectional area? How do their radii compare? (6 Marks)

(ii) If charges flow very slowly through a metal, why does it not require several hours for a light to come on when the switch is thrown? (2 Marks)

b) A copper wire 1.0 mm in diameter and 10 m long is made into a resistor. When a voltage of 1.5 V is applied, the resistor temperature is 600° C.

(i) What is the current in the resistor? (6 Marks)

(ii) What is the power dissipated in the resistor? (3 Marks)

c) The terminal voltage of a battery drawing a current of 5 A is 11 V and when drawing 10-A current is 10 V. What is the emf and the internal resistance of the battery? (8 Marks)

Question 4

a) (i) Suppose an electron is chasing a proton, up along this page, when suddenly a magnetic field is formed perpendicular to the page. What will happen to the particles? (3 Marks)

(ii) How can the motion of a moving particle be used to distinguish between a magnetic field and an electric field? Give a specific example to justify your answer. (4 Marks)

b) An electron in a TV camera tube is moving at 7.20×10^6 m/s in a magnetic field.

(i) Without knowing anything about the field direction, find the greatest and the least magnitudes of the force the electron could feel due to the field? (6 Marks)

(ii) At one point the acceleration of the electron is 4.90×10^{14} m/s^2. What is the angle between the electron's velocity and the magnetic field? Take the magnitude of the field B to be 83.0 mT. (5 Marks)

c) A straight , horizontal stretch of copper wire carries a current i=28 A, moving from left to right. What is the magnitude and direction of the magnetic field B needed to 'float', that is, to balance its weight? Its linear density is 46.4 g/m. (7 Marks)

Question 5

a) A circular loop is located in a uniform and constant magnetic field. Describe, quantitatively, how an emf can be induced in the loop in this situation. (4 Marks)

b) A bar slides to the left without friction along a pair of parallel rails at a constant speed of 5.0 m/s. The magnetic field points out of the paper and is constant at 0.10 T. Assume that the bar and the rail it slides on have negligible resistance. The distance between the rails is 10 cm. A 100-Ω resistor is connected across the right hand side ends of the rails.

(i) Give a sketch of what is depicted above with appropriate labels. (3 Marks)

(ii) Calculate the current through the 100 ohm resistor and give its direction. (7 Marks)

c) A large circular loop of wire lies in the horizontal plane. A bar magnet is dropped through the loop. If the axis of the magnet remains horizontal as it falls, describe the emf induced in the loop. How is the situation changed if the axis of the magnet remains vertical as it falls? (4 Marks)

d) The current in a solenoid is increasing at the rate of 10 A/s. The cross sectional area of the solenoid is π cm^2 and there are 300 turns on its 15-cm length.

(i) What is the induced emf which acts to oppose the increasing current? (4 Marks)

(ii) Keeping the rate of change of current the same, if the induced emf is 50 V, what may the radius of the solenoid be? (3 Marks)

Question 6

a) (i) At what frequency would a 6.0-mH inductor and a 10.0-μF capacitor have the same reactance? (3 Marks)

(ii) What would this reactance be? (2 Marks)

(iii) Show that this frequency would be equal to the natural frequency of the free LC oscillations? (Hint: Start from the relation that in a free oscillator, charge varies with time as

$$L\frac{d^2q}{dt^2}+\frac{q}{C}=0\ )$$

(4 Marks)

b) A series ac circuit contains the following components: R=150 Ω, L=250 mH, C=2μF and a generator with V_m=210 V operating at 50 Hz.

(i) Calculate the inductive reactance. (2 Marks)

(ii) Find the capacitive reactance. (2 Marks)

(iii) Determine the peak current. (6 Marks)

(iv) Evaluate the phase angle. (4 Marks)

(v) What is the power factor? (2 Marks)

SOME USEFUL CONSTANTS

Resistivity of copper = $1.72 \times 10^{-8}\ \Omega.m$
Temperature coefficient of resistivity of copper = 0.00393 /°C
Charge of electron = 1.6×10^{-19} C
Mass of electron = 9.11×10^{-31} kg
Permeability of free space, μ_0 = $4\pi \times 10^{-7}$ T.m/A
Permittivity of free space, ϵ_0 = $8.85 \times 10^{-12}\ C^2.N/m^2$

Some Fundamental Constants[a]

Quantity	Symbol	Value[b]
Atomic mass unit	u	$1.660\ 540\ 2(10) \times 10^{-27}$ kg $931.434\ 32(28)$ MeV/c^2
Avogadro's number	N_A	$6.022\ 136\ 7(36) \times 10^{23}$ (g mol)$^{-1}$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.274\ 015\ 4(31) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k}$	$0.529\ 177\ 249(24) \times 10^{-10}$ m
Boltzmann's constant	$k = R/N_A$	$1.380\ 658(12) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$2.426\ 310\ 58(22) \times 10^{-12}$ m
Deuteron mass	m_d	$3.343\ 586\ 0(20) \times 10^{-27}$ kg $2.013\ 553\ 214(24)$ u
Electron mass	m_e	$9.109\ 389\ 7(54) \times 10^{-31}$ kg $5.485\ 799\ 03(13) \times 10^{-4}$ u $0.510\ 999\ 06(15)$ MeV/c^2
Electron-volt	eV	$1.602\ 177\ 33(49) \times 10^{-19}$ J
Elementary charge	e	$1.602\ 177\ 33(49) \times 10^{-19}$ C
Gas constant	R	$8.314\ 510(70)$ J/K·mol
Gravitational constant	G	$6.672\ 59(85) \times 10^{-11}$ N·m^2/kg^2
Hydrogen ground state	$E_0 = \frac{m_e e^4 k^2}{2\hbar^2} = \frac{e^2 k}{2a_0}$	$13.605\ 698(40)$ eV
Josephson frequency-voltage ratio	$2e/h$	$4.835\ 976\ 7(14) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \frac{h}{2e}$	$2.067\ 834\ 61(61) \times 10^{-15}$ Wb
Neutron mass	m_n	$1.674\ 928\ 6(10) \times 10^{-27}$ kg $1.008\ 664\ 904(14)$ u $939.565\ 63(28)$ MeV/c^2
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	$5.050\ 786\ 6(17) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2 (exact)
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 817 \times 10^{-12}$ C^2/N·m^2 (exact)
Planck's constant	h $\hbar = h/2\pi$	$6.626\ 075(40) \times 10^{-34}$ J·s $1.054\ 572\ 66(63) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\ 623(10) \times 10^{-27}$ kg $1.007\ 276\ 470(12)$ u $938.272\ 3(28)$ MeV/c^2
Quantized Hall resistance	h/e^2	$25812.805\ 6(12)\ \Omega$
Rydberg constant	R_H	$1.097\ 373\ 153\ 4(13) \times 10^7$ m^{-1}
Speed of light in vacuum	c	$2.997\ 924\ 58 \times 10^8$ m/s (exact)

[a] These constants are the values recommended in 1986 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see Cohen, E. Richard, and Barry N. Taylor, *Rev. Mod. Phys.* 59:1121, 1987.
[b] The numbers in parentheses for the values below represent the uncertainties in the last two digits.