

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF PAPER: MODERN PHYSICS & WAVE OPTICS

COURSE NUMBER: P231

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT
HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

THE LAST PAGE CONTAINS FORMULAE AND CONSTANTS THAT MAY BE
USEFUL IN SOME PROBLEMS

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE
CHIEF INVIGILATOR

QUESTION 1

- (a) List the four features of the photoelectric effect that do not agree with classical theory. **(4 marks)**
- (b) In an experiment to determine the maximum kinetic energy of ejected photoelectrons a metallic sodium surface was illuminated with light of wavelength 300 nm. The work function of sodium is 2.46 eV.
- (i) Find the kinetic energy of the ejected photoelectrons. **(3 marks)**
 - (ii) What is the cutoff frequency of sodium? **(3 marks)**
 - (iii) What is the maximum speed of the ejected photoelectrons? **(2 marks)**
- (c) X-rays of wavelength of $\lambda = 0.20$ nm are scattered by a block of material and the scattered waves are observed at 45° to the incident beam.
- (i) Calculate the wavelength of the scattered X-rays. **(3 marks)**
 - (ii) What is the kinetic energy of the scattered electrons if they were initially at rest? **(3 marks)**
- (d) For Compton scattering of electrons, derive an expression that can enable you to calculate the recoil angle of the scattered electron in terms of the incident wavelength λ and scattering angle θ of the X-rays, and Compton shift wavelength of the X-rays λ' . **(7 marks)**

QUESTION 2

(a) Explain the four ideas of Bohr from which he derived the Bohr atomic model. **(8 marks)**

(b) Consider a single electron orbiting a stationary nucleus with charge $+Ze$, where e is the charge of single proton and Z is the atomic number.

(i) What is the total energy of the system using a classical model? Clearly show with the aid of a diagram how you obtain your result. **(4 marks)**

(ii) Show that the n^{th} radius of orbit of the electron is given by

$$r_n = n^2 \frac{a_0}{Z},$$

Where $a_0 = \frac{\hbar^2}{m_e k_e e^2}$ is the Bohr radius. **(8 marks)**

(c) Use the Rydberg equation to calculate the wavelength of the radiation emitted by a hydrogen atom from $n = 273$ to $n = 272$ levels. **(2 marks)**

(d) Use classical mechanics to calculate the wavelength of the radiation at the $n = 273$ orbit, where its radius of orbit is $r = 3.94 \mu\text{m}$ and its velocity is $v = 8.01 \times 10^3$ m/s. **(3 marks)**

QUESTION 3

(a) Discuss the properties of the three particles produced from natural radioactivity. Also state how each can be detected. **(6 marks)**

(b) Starting with the equation

$$\frac{dN}{dt} = -\lambda N,$$

where N is the instantaneous quantity radioactive atoms in the sample, t is the time and λ is the decay constant, show that the time it takes for half the sample to decay to half its original quantity is

$$T_{1/2} = \frac{\ln 2}{\lambda}. \quad \textbf{(6 marks)}$$

(c) Calculate the Q value for the reaction ${}^1_1\text{H} + {}^{19}_9\text{F} \rightarrow {}^{16}_8\text{O} + {}^4_2\text{He}$ in MeV. Use full calculator accuracy and round at the end to 3 significant figures. What kind of a reaction is this in terms of energy? **(4 marks)**

(d) Neutron capture.

- (i) What is a thermal neutron? Calculate its average energy for a neutron at room temperature $T_{\text{room}} = 300$ K. **(3 marks)**
- (ii) Consider a neutron with an energy above 1 MeV traveling through matter. Discuss how such a neutron can be captured and write down the equation for neutron capture and explain it. **(6 marks)**

QUESTION 4

(a) Figure 1. Illustrates the Young double slit experiment.

(i) Show that the distance between successive maxima is given by

$$y_{m+1} - y_m = \frac{\lambda L}{d}.$$

(8 marks)

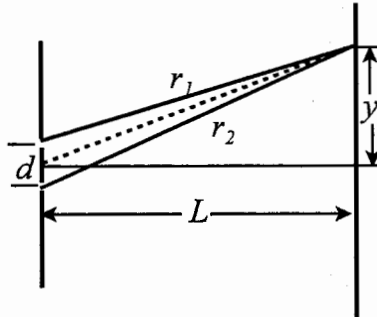


Figure 1.

(ii) If a wavelength of 589 nm is used where $L = 2\text{m}$ and $d = 0.05\text{ m}$, find the distance of separation between successive maxima. (2 marks)

(b) (i) Find the grating constant for a grating if the second order spectrum is observed at an angle of 16° using the neon laser at 633 nm. (4 marks)

(ii) For the same grating as in (i), what wavelength is observed at 16° in the third order? (2 marks)

(c) (i) Show that if unpolarised light is incident to a polariser aligned along the z -axis and the light travels along the y -axis the intensity is reduced by half. (6 marks)

(ii) If a second polariser is placed after the first at an angle of 60° with the z -axis, how much light goes through the second polariser compared to the incident unpolarised light. (3 marks)

QUESTION 5

- (a) Discuss in detail how a photomultiplier tube produce a large current pulse from an incident photon. **(8 marks)**
- (b) Discuss the origins of characteristic and continuous optical spectra from excited atoms. **(8 marks)**
- (c) Explain the process of stimulated emission of radiation. **(4 marks)**
- (d) The Helium-neon laser operating at 632.8 nm has a cavity of length 20 cm. Determine the number of vibrational modes in the cavity. Show clearly how you obtain your solution. **(5 marks)**

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

$$\sigma = 5.669\ 6 \times 10^{-8} \text{ W}/(\text{m}^2\text{K}^2)^3$$

Boltzmann's constant, $k_B = 1.3801 \times 10^{-23} \text{ J/K}$

Bohr magneton, $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$

Bohr radius $a_0 = 0.0529 \text{ nm}$

Rydberg constant $R_H = 1.097\ 373\ 2 \times 10^7 \text{ m}^{-1}$

Speed of light in vacuum, $c = 2.997\ 924\ 58 \times 10^8 \text{ m/s}$

Planck's constant, $h = 6.626\ 075 \times 10^{-34} \text{ Js}$

$$\hbar = 1.054\ 572 \times 10^{-34}$$

$$hc = 1.986\ 447 \times 10^{-25}$$

$$2\pi\hbar c^2 = 3.741\ 859 \times 10^{-15}$$

mass of an electron, $m_e = 9.109\ 389\ 7 \times 10^{-31} \text{ kg}$

mass of a proton, $m_p = 1.672\ 623 \times 10^{-27} \text{ kg}$

mass of a neutron, $m_n = 1.674\ 928\ 6 \times 10^{-27} \text{ kg}$

Coulomb constant, $k_e = 8.987\ 551\ 787 \times 10^9 \text{ Nm}^2/\text{C}^2$

electron charge, $e = 1.602\ 177\ 33 \times 10^{-19} \text{ C}$

1 atomic mass unit = 1 amu = 1 u = $1.660\ 540\ 2 \times 10^{-27} \text{ kg} = 931.494 \text{ MeV}$

1 eV = $1.602\ 177\ 33 \times 10^{-19} \text{ J}$

$T_{1/2}({}^{14}\text{C}) = 5730 \text{ years}$

$$\frac{N({}^{14}\text{C})}{N({}^{12}\text{C})} = 1.3 \times 10^{-12}$$

Hydrogen (${}^1_1\text{H}$) mass 1.007 825

Helium (${}^4_2\text{He}$) mass = 4.002 602 u

Oxygen (${}^{16}_8\text{O}$) mass = 15.994 915 u

Fluorine (${}^{19}_9\text{F}$) mass = 18.998 404 u

$$\lambda_{\max} = \frac{hc}{4.965kT}$$

$$I = \frac{2\pi\hbar c^2}{\lambda^5 \left(e^{\frac{\hbar c}{\lambda k}} - 1 \right)}$$

$$\int \cos^2 u du = \frac{\lambda^5}{2} + \frac{\lambda^5}{4} \sin 2u$$

$$\theta_{\min} = 1.22\lambda/D$$

$$\Delta \nu = \frac{c}{2L}$$