UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF PAPER	:	THERMODYNAMICS
COURSE NUMBER	:	P242
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

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INFORMATION

For a monatomic gas: $\gamma = \frac{5}{3}$ and $C_V = \frac{3}{2}R$

For a diatomic gas: $\gamma = \frac{7}{5}$

Universal gas constant = 8.31 J $mol^{-1}K^{-1}$

Specific heat of water = 4200 J $kg^{-1}K^{-1}$

Avogadro's number = 6.02×10^{23} mol^{-1}

QUESTION 4

(a) One mole of a monatomic, ideal gas initially at temperature T_o, expands from volume V_o to $2V_o$,

(i) at constant temperature, (5 marks)
(ii) at constant pressure. (5 marks)

Determine expressions for the work done during expansion and the heat absorbed by the gas in each case.

(b) 10 litres of gas at atmospheric pressure is compressed isothermally to a volume of 1 litre and then allowed to expand adiabatically to 10 litres.

(i) Sketch the processes on a pV diagram for a monatomic gas. Justify the sketch. (9 marks)
(ii) Make a similar sketch for a diatomic gas. Justify the sketch. (4 marks)
(iii) Is a net work done on or by the system? (2 marks)

QUESTION 1

(a) Show that for an ideal gas with constant heat capacities the slope of an adiabatic curve is negative and that it has a larger absolute value than the slope of an isothermal curve at the same values of p and V. Assume that the process is quasistatic. (10 marks)

(b) A Carnot engine has a cycle pictured in Fig. 1.1

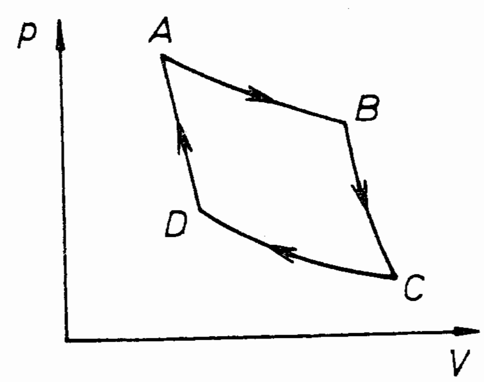

Fig. 1.1

(i) What thermodynamic processes are involved at boundaries *AD* and *BC*; *AB* and *CD*? (2 marks)

(ii) Where is work put in and where is it extracted? (2 marks)

(iii) If the above is a steam engine with $T_H = 450\ K$, operating at room temperature (300 K), calculate the efficiency. (2 marks)

(c) To maintain a freezer box at -40°C on a summer day when the ambient temperature is 27°C, heat is removed at the rate of 1.25 kW. What is the coefficient of performance of the freezer, and what is the power that must be supplied to the freezer? (4 marks)

(d) One kilogramme of water at 0°C is brought in contact with a heat reservoir at *100°C*. When the water has reached *100°C*, what is the change in entropy of the water? (5 marks)

QUESTION 3

(a) Write down the van der Waals' equation of state and discuss its meaning, briefly, with reference to the ideal gas law.

(9 marks)

(b) Fig. 3.1 shows isotherms of CO_2. The isotherm corresponding to T = 304 K is called the critical isotherm. It is distinguished by having a minimum and a point of inflection that coincide at a single point.

Derive expressions for the van der Waals constants 'a' and 'b' in terms of the critical pressure, p_{cr}, and the critical temperature, T_{cr}.

(8 marks)

(c) The van der Waals equation of state can be expressed in virial form using the following equation:

$$pv = RT\left(1 - \frac{b}{v}\right)^{-1} - \frac{a}{v}.$$

With the aid of the binomial theorem, determine the first and second virial coefficients.

(8 marks)

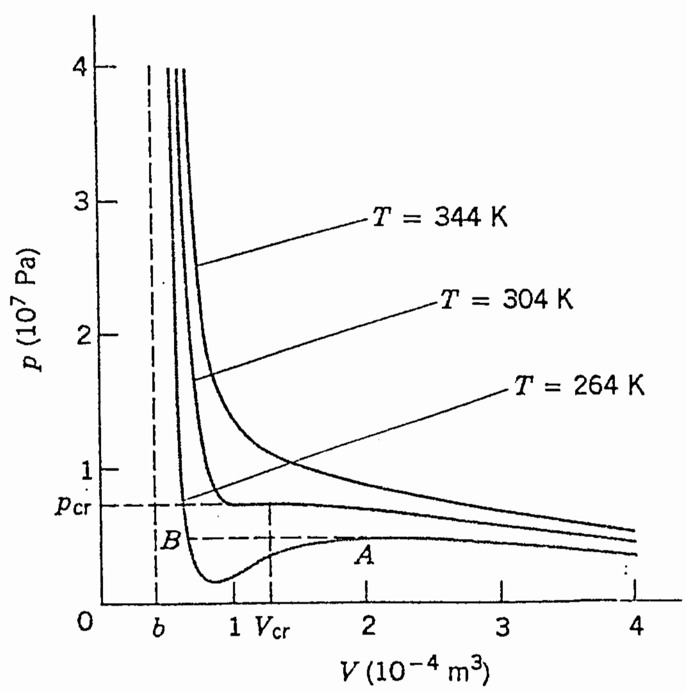

Fig. 3.1

QUESTION 5

(a) Consider an ideal gas in a container such as the one shown in Fig. 5.1. The pressure, p exerted by the gas onto the area AB is given by the following equation:

$$\frac{m\left[\left(v_x^{\,2}\right)_1+\left(v_x^{\,2}\right)_2+\ldots\ldots\ldots\ldots\ldots\ldots+\left(v_x^{\,2}\right)_N\right]}{ABL}$$

Show that the root-mean-square speed V_{rms} of the gas molecules is given by $(3p/\rho)^{½}$. where ρ is the density of the gas.

(N is the number of molecules in the container, m is the mass of a molecule and v_x is the speed of a molecule in the x-direction). (14 marks)

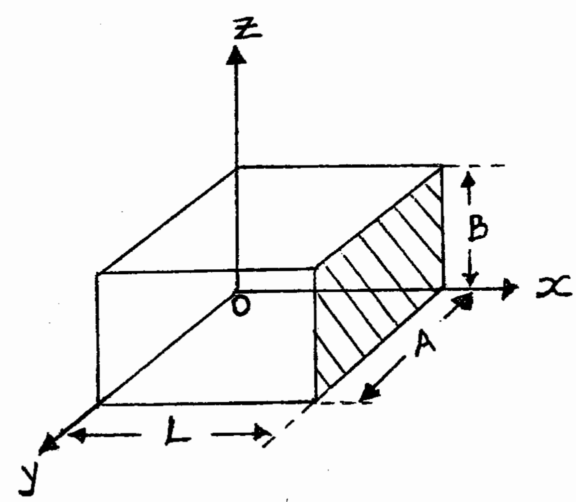

Fig. 5.1

(b) Measurements by spacecraft have shown that near the surface of the planet Venus, the atmospheric pressure is 90 times that on the Earth's surface, and the temperature is 500°C, compared to typical temperatures on Earth of 10°C.

What is the ratio of the speed of an average carbon dioxide molecule on Venus to that of an average carbon dioxide molecule on Earth? (4 marks)

(c) Calculate the magnitude of the mean free path and the collision frequency of air molecules at 0°C and 1 atmospheric pressure. Note that the effective molecular radius is 1 Å. For the conditions stated, the average speed of air molecules is about 1×10^7 m/s and there are about 3×10^{25} molecules/m^3. (7 marks)

QUESTION 2

(a) Consider a water pipe of internal radius x, external radius y and length z. The inside temperature is T_1 while the surroundings are at a temperature of T_2 (where $T_1 > T_2$). Show that heat is conducted through the walls of the pipe at the rate

$$\frac{dQ}{dt} = \frac{-2\pi k\left(T_2 - T_1\right)z}{\ln\left(\frac{y}{x}\right)}.$$

[k is the thermal conductivity of the pipe]. (12 marks)

(b) A copper bar which is 1.6 m long and has a circular cross-section that is 0.01 m in diameter, is initially at room temperature. It is placed end-to-end between a heat reservoir at 20°C and one at 80°C. Before steady state is reached, the heat flow in the copper bar is described by the following equation:

$$\frac{dQ}{dt} = 7.6 - 0.5\exp(-t),$$

where Q is the heat in joules and t is the time in seconds

Assuming that heat started flowing at time t = 0, determine how much heat entered the bar after 2 s. (5 marks)

(c) The surface temperature of a spherical mass of molten metal (the source), 2.5 m in radius, is 800°C. It is surrounded by a spherical shell of inside radius 2.5 m, outside radius 5.0 m, and thermal conductivity 35 J/m.C°.s.If the outside of this sphere is exposed to room temperature (25°C), find the thermal power coming from the source. (8 marks)