UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE
DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF THE PAPER: **COMPUTATIONAL PHYSICS -I**

COURSE NUMBER : **P262**

TIME ALLOWED : **THREE HOURS**

SECTION A: ONE HOUR.
SECTION B: TWO HOURS.

INSTRUCTIONS:

SECTION A **: THIS IS THE WRITTEN PART TO BE COMPLETED IN YOUR ANSWER BOOK. CARRIES A TOTAL OF <u>30</u> MARKS.**

SECTION B**: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF <u>70</u> MARKS.**

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS <u>FOUR</u> PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section) **Time: One Hour**

Q.1.:

(A) Explain the difference between [5]

(i) ***y1:= a*x +b*** and ***y2:= (a,b,x) → a*x+b*** .
(ii) ***diff*** and ***Diff*** .
(iii) ***solve*** and ***fsolve*** .
(iv) ***Pi*** and ***pi*** .
(v) the symbol **( )** and **[ ]** .

(B) What will be the result of entering following Maple statements. [3]

(i) **a:= 1 / 4** and **b:= 1.0 / 4** .
(ii) **seq(2*n^3 , n=1..4)**
(iii) **end;**

(C) Write Maple statements to calculate [2]

(i) $\int_0^2 x^3 dx$

(ii) $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\frac{1}{7^2}$

(D) Write Maple statements to produce 10 pairs of data sets (x_i , y_i), i = 1 . . 10 using the following relation . [5]

$$y_i = 0.3 + 0.2 * x_i + 1.5 * sin(x_i)$$

for $0 \le x \le 2.0$.

Q.2. The steady state temperature distribution T(x) along a particular thin bar of unit length is given by

$$\frac{d^2T}{dx^2} = 10(T-20)$$

The boundary conditions are at x=0, T=100^0 C and $\frac{dT}{dx} = 1.0$.

Using the Maple commands:

(i) Find the exact solution for T(x). Plot the solution for the interval $0 \le x \le 1$. [4]

(ii) Solve the equation numerically using default numerical method available in Maple. Plot the solution in the interval $0 \le x \le 1$. [4]

(iii) Plot solutions of (i) and (ii) on one graph. [2]

(iv) Convert the second order differential equation into two first order [5] differential equations. Solve the two first order differential equations numerically using default numerical method available in Maple.

Q.3. (a) Using Maple statements generate 10 data points (x_i, y_i) using the equation [5]

$$y_i = 0.1 + 9.8\,x_i$$

for $x_i = x_{i-1} + \Delta x$ for $i = 0...10$ with $x_0 = 1.0$ and $\Delta x = 0.1$.

(b) Using the above data points, write a program in Maple to calculate [10]

$$\sum_{i=0}^{10} x_i \;,\sum_{i=0}^{10} y_i \;,\sum_{i=0}^{10} x_i^2 \;,\sum_{i=0}^{10} y_i^2 \;,\sum_{i=0}^{10} x_i y_i \;, \text{and} \sum_{i=0}^{10} (x_i^2 - x_i y_i)$$

SECTION B
(Practical Section)

Time: Two Hours.

Q.4. (a) Plot the function $f(x) = sin^2(x) - x^3$ for x = -1 to 2. [5]
How many roots are there?

(b) Solve the equation $f(x)=0$ to determine its roots. [10]

Q.5. Find the solution of the initial value problem [10]

$$\frac{dy}{dx} = y\,cot(x) + 20\,sin(x)\,cos(20x)$$

with initial boundary condition $y(\pi/2) = 0$..
Plot the solution. Explain why there is beat phenomena. [5]

Q.6. The velocity v(t) of a falling body of unit mass under the influence of air resistance from a height is described by the differential equation

$$\frac{dv(t)}{dt} = -9.8 + 0.009v^2(t)$$

Initial value is given for t=0 as v(0)=0.

(A) Solve the equation for v(t) using Maple commands.
Plot v(t) vs t for t=0..15. [10]

(B) At time t=10 , a parachute is opened, resulting into a differential equation for the fall as

$$\frac{dv(t)}{dt} = -9.8 + 0.1v^2(t)$$

(i) Convert the equation of (B) into a differential equation for distance x(t). [5]
Initial boundary condition is at time t=10 and this is
x(10)= 500
v(10)= value of v(t) from (i) at time t=10.

(ii) Solve the differential equation numerically using default method of Maple. [10]

(iii) Plot x(t) vs t for t=10..100. [2]

(iv) From the graph estimate the time t when the falling body touches ground. [5]

(v) From this time find x(t) and v(t). [8]

@@@ END OF EXAMINATION @@@