UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF PAPER	:	ELECTROMAGNETIC THEORY
COURSE NUMBER	:	P331
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 11 PAGES, INCLUDING THIS PAGE.

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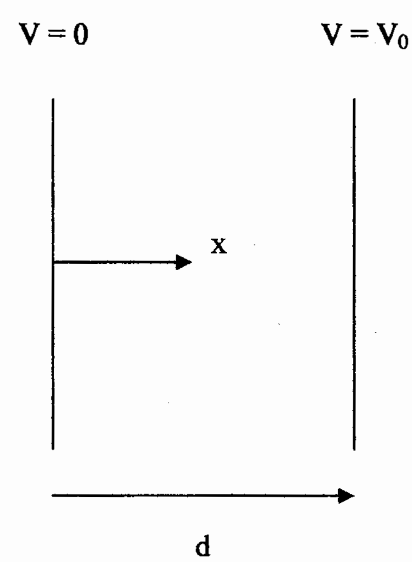
Question one

Starting from

$$\int_{area} \mathbf{E}.\mathbf{da} = \frac{Q}{\varepsilon_0}$$

where Q is the total charge contained within volume V, and the integral involves the flux of e through the surface enclosing v, derive the equations of Poisson and Laplace. (10 marks)

Two infinite parallel plates separated by a distance d are at potentials 0 and V_0 respectively (see figure). The space charge density between the plates varies as $\rho = \rho_0 x/d$. x is a distance measured from the plate at zero potential. Determine how the potential varies between the plates. (15 marks)



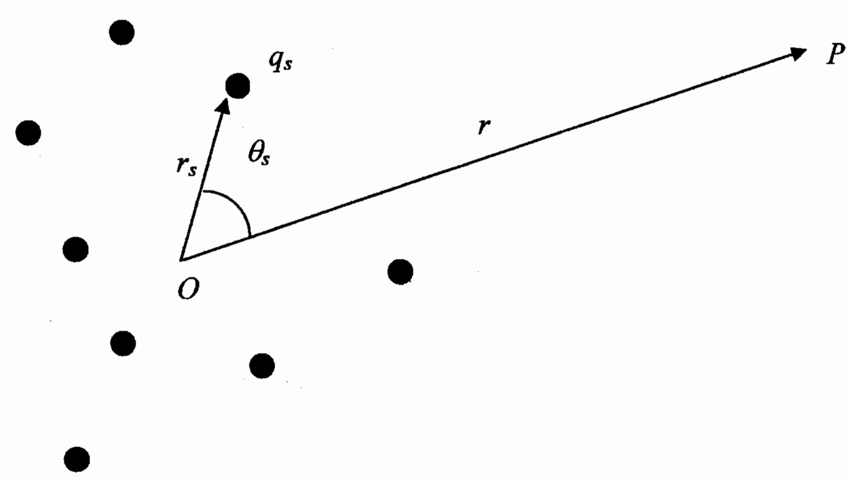
Question two

What is the principle of superposition? (2 marks)

Show that the electric potential far away from an arbitrary distribution of stationary charges $q_1, q_2, \ldots\ldots, q_s, \ldots\ldots$ is given by:

$$V(r) = \left(\frac{1}{4\pi\varepsilon_0 r}\right)\sum_s \left[1 + \cos\theta_s\left(\frac{r_s}{r}\right) + \frac{1}{2}\left(3\cos^2\theta_s - 1\right)\left(\frac{r_s}{r}\right)^2 + \ldots\ldots\right] q_s,$$

where r_s is the distance of charge q_s from the origin, r the distance from the origin where the potential is evaluated, at point P, and θ_s is the angle between r and r_s, as shown in the diagram.



(10 marks)

Show that the magnitude of the dipole moment of the charge distribution is independent of the choice of origin if the charge distribution is electrically neutral, but can be made to be equal to zero if the distribution is not electrically neutral. (5 marks)

Consider two equal and opposite charges of magnitude q separated by a small, fixed distance ℓ in an electric field $\boldsymbol{E}$. Show that the dipole experiences a translational force given by $(\mathbf{p}.\nabla)\mathbf{E}$ where $\boldsymbol{p}$ is the dipole moment equal to $q\boldsymbol{\ell}$ with $\boldsymbol{\ell}$ a vector of length ℓ and direction from $-q$ to +q.

(6 marks)

Under what conditions is the magnitude of this force zero? (4 marks)

Question three

A simple model for a conductor supposes that the current is carried by mobile electrons that, on average, suffer a collision every τ seconds as they travel through the conductor. Show that this model, with suitable assumptions, leads to an expression for a frequency-dependent conductivity, $\sigma(\omega)$, given by

$$\sigma(\omega) = \frac{Ne^2\tau}{m[1 + j\omega\tau]},$$

where N in the number density of mobile electrons. List all the assumptions you have made in deriving this expression. (10 marks)

Find the high-frequency limit for $\sigma(\omega)$ and comment on your result.

(5 marks)

Derive an approximate expression for the phase angle between E and H in a conductor where $\frac{\sigma}{\omega\varepsilon} \leq 0.1$. (10 marks)

Question four

Consider a plasma in which the electrons are free to move and the motion of the more massive positively charged ions can be neglected. Show that the wavevector, k, for a plane electromagnetic wave propagating in such a medium is given by the expression

$$k = k_0\left[1-\left(\frac{\omega_P}{\omega}\right)^2\right]^{\frac{1}{2}}.$$

k_0 is the wavenumber in free space, and ω_P is the so-called plasma angular frequency. $\omega_P = \left[\frac{Ne^2}{\varepsilon_0 m}\right]^{\frac{1}{2}}$ where N is the number of free electrons per cubic metre and m is the mass of an electron. (8 marks)

Use this expression to determine

a) the product of the phase and group velocities

b) the magnitude of the total current density when $\omega = \omega_P$

(6 marks)

Obtain an expression for the wavenumber of an electromagnetic plane wave in the plasma for frequencies very much less than the plasma frequency. Hence determine the distance in which the amplitude of a very low-frequency wave has decayed to 1/e of its value on entering a plasma in which $\omega_P = 10^{12}$ secs^{-1}. (6 marks)

In the ionosphere at an altitude between about 100 and 150 km, the electron density increases with height. Hence show that an electromagnetic wave propagating away from the Earth in this region is bent away from the normal to the Earth's surface. (5 marks)

Question 5

Starting from Maxwell's equations (see attached sheet of useful information), show that the square of the wavevector of an infinite electromagnetic plane wave propagating in a linear, homogeneous medium, is given by:

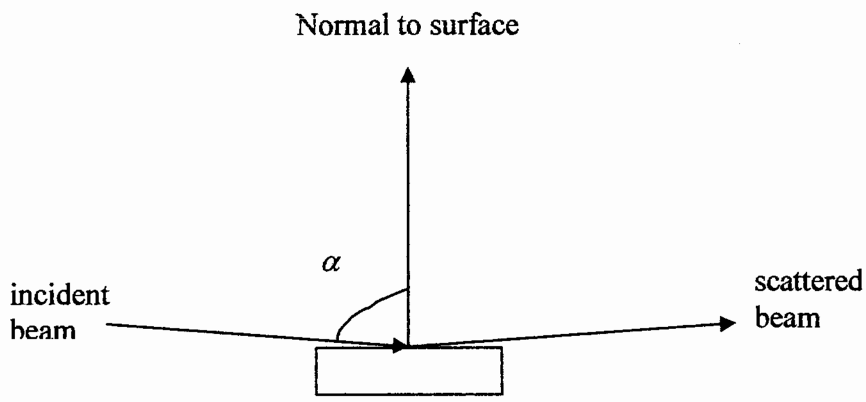
$$k^2 = \varepsilon_r \mu_r k_0^2 \left[1 - \frac{j\sigma}{\omega \varepsilon_r \varepsilon_0} \right],$$

where k_0 is the magnitude of the wavenumber in free space, ε_r and μ_r are the relative permittivity and permeability of the medium, σ its electrical conductivity and ω the angular frequency of the wave. (10 marks)

The equation given above implies that the refractive index, n, defined as k/k_0, in a non-magnetic insulator should have a value equal to $\varepsilon_r^{1/2}$. However, for water its value at optical frequencies is about 1.3, while ε_r measured at low frequencies is about 81. Explain why these two values are different.

(5 marks)

An experimental technique, known as grazing incident angle X-ray scattering, is often used to study the structure of thin films (often thin superconducting films) deposited onto single crystal substrates. The angle α, between the normal to the surface of the thin film and parallel incident beam is varied from nearly 90° to about 85° during the experiment. The intensity of the scattered beam for values of α close to 90° is equal to the incident beam intensity. Explain why this occurs.



(5 marks)

In one experiment, the intensity of the scattered beam was equal to the intensity of the incident beam until the value of α was less than 89.3°. What is the refractive index of the thin film at X-ray frequencies? (5 marks)

PHM31E ELECTROMAGNETISM & RELATIVITY

Important equations required to describe the propagation of electromagnetic radiation in matter.

1 $\nabla.(\varepsilon\mathbf{E}) = \rho_f$

2 $\nabla.\mathbf{H} = 0$

3 $\nabla\mathbf{x}\mathbf{E} + \mu\dot{\mathbf{H}} = 0$

4 $\nabla\mathbf{x}\mathbf{H} - \varepsilon\dot{\mathbf{E}} = \mathbf{J}_f = \sigma\mathbf{E}$

$\nabla\mathbf{x}\nabla\mathbf{x}\mathbf{E} = -\nabla^2\mathbf{E} + \nabla(\nabla.\mathbf{E})$ cartesian co-ordinates

5 $\nabla^2\mathbf{E} - \mu\varepsilon\ddot{\mathbf{E}} - \mu\sigma\dot{\mathbf{E}} = 0$

6 $\nabla^2\mathbf{H} - \mu\varepsilon\ddot{\mathbf{H}} - \mu\sigma\dot{\mathbf{H}} = 0$

7 $\mathbf{E} = \mathbf{E_0} \exp j(\omega t - kz)\,\mathbf{i}$

8 $\mathbf{H} = (k/\omega\mu)\mathbf{E_0} \exp j(\omega t - kz)\,\mathbf{j}$

9 $-k^2 + \omega^2\varepsilon\mu - j\omega\sigma\mu = 0$

10 $k^2 = \varepsilon_r\mu_r k_0^2[1 - j\sigma/\omega\varepsilon]$

$k_0 = \omega/c$

$c = (\mu_0\varepsilon_0)^{-½}$

$\mathbf{S} = \mathbf{E}\mathbf{x}\mathbf{H}$ Poynting's vector

$E/H = \omega\mu/k$ wave impedance

PHYSICAL CONSTANTS AND UNITS

Acceleration due to gravity	g	$9.81\ m\ s^{-2}$
Gravitational constant	G	$6.673 \times 10^{-11}\ N\ m^2\ kg^{-2}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ mol^{-1}$
	(Note: 1 mole = 1 *gram* molecular-weight)	
Ice point	T_{ice}	273.15 K
Gas constant	R	$8.314\ J\ K^{-1}\ mol^{-1}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ J\ K^{-1} = 0.862 \times 10^{-4}\ eV\ K^{-1}$
Stefan constant	σ	$5.670 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$
Rydberg constant	R_∞ $R_\infty hc$	$1.097 \times 10^7\ m^{-1}$ 13.606 eV
Planck constant	h	$6.626 \times 10^{-34}\ J\ s = 4.136 \times 10^{-15}\ eV\ s$
$h/2\pi$	$\hbar$	$1.055 \times 10^{-34}\ J\ s = 6.582 \times 10^{-16}\ eV\ s$
Speed of light *in vacuo*	c	$2.998 \times 10^8\ m\ s^{-1}$
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	$8.854 \times 10^{-12}\ F\ m^{-1}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ H\ m^{-1}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ A\ m^2\ (J\ T^{-1})$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ A\ m^2\ (J\ T^{-1})$
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_c = h/mc$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
torr (mm Hg, 0°C)	torr	$133.32\ Pa\ (N\ m^{-2})$
barn	b	$10^{-28}\ m^2$

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