

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF THE PAPER: NUCLEAR PHYSICS

COURSE NUMBER : P442

TIME ALLOWED : THREE HOURS

INSTRUCTIONS:

- ANSWER ANY **FOUR** OUT OF **FIVE** QUESTIONS.
- EACH QUESTION CARRIES **25** MARKS.
- MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION GIVEN IN THE ATTACHED **APPENDIX** WHEN NECESSARY.

THIS PAPER HAS **SEVEN** PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

Q.1:

(A) State whether each of the following statements is true or false. Give reasons, where applicable. [5]

(i) Electron scattering experiments determine directly the nuclear density distribution.

(ii) Both members of a mirror-nuclei pair have the same number of protons.

(iii) The fact that there are no bound states of the di-neutron and di-proton means that the nuclear force between a neutron and a proton is much stronger than that between two neutrons or between two protons.

(iv) Free electrons cannot exist in the nucleus.

(v) Even-Even nuclei in their ground states always have the angular momentum and parity equal to 0^+ .

(B)

(i) Write the general form of nuclear reaction (d , p). [1]

(ii) Express nuclear reaction Q in terms of mass excess in this case. [2]

(iii) In $^{56}\text{Fe}(d,p)^{57}\text{Fe}$ reaction, calculate Q-value. [2]

(C) Describe the three significant processes through which the γ -rays in the energy range 10 keV to 10MeV primarily interact with matter. [5]

(D) Write short notes on [10]

(i) Fluorescent radiation.

(ii) Auger process.

(iii) Nuclear Fission.

(iv) Internal Conversion.

(v) Bremsstrahlung.

Q.2.

(A) Describe briefly the main properties of nuclear forces as learned from a study of two nucleon problems. [10]

(B) Using harmonic oscillator potential of the form

$$V(r) = -V_0 \left[1 - \left(\frac{r}{R} \right)^2 \right] \text{ for all } r$$

where R is the range of the potential, the energy eigen values for the shell model are given by

$$E_{nl} = (2n + \ell - \frac{1}{2}) \hbar \omega$$

where $n=1,2,3\dots$ and $\ell=0,1,2,\dots,n-1$ =orbital angular momentum.

(i) Is the potential given a central potential? [1]

(ii) Is the orbital angular momentum conserved? [1]

(iii) Draw the energy level diagram up to $n=3$. Indicate the states in the notation s, p , etc. [5]

(iv) What do you understand by the term "Magic Numbers" ? [1]

(v) How many magic numbers can you get in this case? [2]

(C) It has been found that the α -emission of the Thorium isotopes can be described by the relation [5]

$$\log_{10}(\lambda) = 56.13 - \frac{105.07 \times 10^7}{v_\alpha}$$

where v_α is the velocity of α -particle in m/sec .

The α -energy in the case of ^{224}Th is 7.33 MeV.

What is the half life of ^{224}Th as estimated, from the above relation?

Q.3. Semi-empirical formula for binding energy is given by

$$B(N,Z) = aA - bA^{2/3} - s \frac{(N-Z)^2}{A} - d \frac{Z^2}{A^{1/2}} - \delta \frac{1}{A^{1/2}}$$

with $a = 15.835$ MeV , $b = 18.33$ MeV , $s = 23.20$ MeV, $d = 0.714$ MeV and

$$\begin{aligned} \delta &= 11.2 \text{ MeV} && \text{for odd-odd or even-even} \\ &= 0 && \text{for odd-even or even-odd.} \end{aligned}$$

(i) Derive the condition for the most stable nucleus of a given odd mass number. [10]

(ii) Use this condition to determine the atomic number Z for the β -stable nucleus of mass $A=27$. [5]

(iii) Neglecting the contribution of the term involving δ , show that the Q -value for the symmetric fission, is given by [10]

$$Q = 6.7828 A^{2/3} + 0.2642 Z^2 A^{-1/3}$$

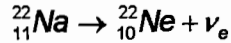
Explain why this Q value should be at least equal or greater than the Coulomb potential between the constituents when they are separated.

Q.4.

(A)

(i) Explain why two types of selection rules (the Fermi and GT-selection rules) exist in β -decay. [2]

(ii) Following is the disintegration process involving electron capture [5]



Assuming Na to be at rest, establish a relation relating to the momentum of the neutrino and the recoil energy of Ne.

(iii) Explain how the concept of helicity is related to the verification of non-conservation of parity in β -decay under the assumption that that mass of neutrino is zero. [5]

(B)

(i) ${}^7_8\text{Li}$ emits a 0.48 MeV , γ -ray in transition from $\frac{1}{2}^-$ to $\frac{3}{2}^-$. [5]
What are the possible choices for the multi-polarity and nature of emitted radiation.

(ii) Explain why transition from 0^+ to 0^+ will not allow any γ -radiation. [2]

(iii) ${}^{57}\text{Co}$ which has the spin-parity $\frac{7}{2}^-$, decays by an allowed GT-transition through K-Capture to an excited state of ${}^{57}\text{Fe}$. [6]
The following γ -emissions are observed in ${}^{57}\text{Fe}$.

E_γ (MeV)	Multipolarity
0.136	E2 (10%)
0.1216	M1 (90%)
0.0144	M1

The ground state of ${}^{57}\text{Fe}$ is $\frac{1}{2}^-$.

Construct a self-consistent energy level scheme for ${}^{57}\text{Fe}$ and assign spins and parities.

Q.5.:

(A) List the conservation laws in Nuclear Reactions. [5]

(B) (i) In a nuclear reaction involving a projectile of mass M_1 with kinetic energy E_1 and a stationary target of mass M_2 produces products of mass M_3 of kinetic energy E_3 and mass M_4 of kinetic energy E_4 .

Show that Q-value (in the lab.system) is given by [15]

$$Q = E_3 \left(1 + \frac{M_3}{M_4}\right) - E_1 \left(1 - \frac{M_1}{M_4}\right) - 2 \frac{\sqrt{M_1 M_3 E_1 E_3}}{M_4} \cos \theta$$

where θ is the angle between the projectile direction and the outgoing nuclei M_3 .

(ii) In a thermal neutron reaction $^{10}\text{B} (n, \alpha) ^7\text{Li}$, the Q-value is found to be 2.8 MeV. Assuming that kinetic energy of a thermal neutron to be small (i.e. equal to zero), show that kinetic energy of α -particle or ^7Li is same for all the angles θ . [3]

(iii) Show that in the case where $Q < 0$, and $E_1 \rightarrow 0$, there is no reaction. [2]

@@@END OF EXAMINATION@@@

Appendix

Selection Rules:

(A) β -decay:

Type of Transition		ΔI	Parity Change
Allowed	Fermi	0	No
	GT	± 1 or 0 (except $0 \rightarrow 0$)	No
1 st Forbidden	Fermi	$\pm 1, 0$ (except $0 \rightarrow 0$)	Yes
	GT	$\pm 2, \pm 1$, or 0 (except $0 \rightarrow 0$; $1/2 \rightarrow 1/2$; $0 \rightarrow 1$)	Yes

(B) γ - decay:

	E1	E2	E3	E4
$\Delta\pi$	Yes	No	Yes	No
$ \Delta J \leq$	1	2	3	4
	M1	M2	M3	M4
$\Delta\pi$	No	Yes	No	Yes
$ \Delta J \leq$	1	2	3	4

(C) Useful Information

PHYSICAL CONSTANTS ¹ AND DERIVED QUANTITIES

Speed of light $c = 2.99792458 \times 10^8 \text{ m s}^{-1} \sim 3.00 \times 10^{23} \text{ fm s}^{-1}$

Avogadro's number $N_A = 6.02214199(47) \times 10^{26}$ molecules per kg-mole

Planck's constant $h = 6.62606876(52) \times 10^{-34} \text{ J s}$

$$\hbar = 1.054571596(82) \times 10^{-34} \text{ J s} = 0.65821 \times 10^{-21} \text{ MeV s}$$

$$\hbar^2 = 41.802 \text{ u MeV fm}^2$$

$$\hbar c = 197.327 \text{ MeV fm}$$

Elementary charge $e = 1.602176462(63) \times 10^{-19} \text{ C}$

$$e^2/4\pi\epsilon_0 = 1.4400 \text{ MeV fm}$$

Fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = 1/137.036$

Boltzmann constant $k = 1.3806503(24) \times 10^{-23} \text{ JK}^{-1} = 0.8617 \times 10^{-4} \text{ eV K}^{-1}$

MASSES AND ENERGIES

Atomic mass unit m_u or $u = 1.66053873(13) \times 10^{-27} \text{ kg}$

$$m_u c^2 = 931.494 \text{ MeV}$$

Electron $m_e = 9.10938188(72) \times 10^{-31} \text{ kg}$

$$m_e/m_u = 5.486 \times 10^{-4} = 1/1823$$

$$m_e c^2 = 0.510998902(21) \text{ MeV}$$

Proton $m_p = 1.67262158(13) \times 10^{-27} \text{ kg}$

$$m_p/m_u = 1.00727647$$

$$m_p c^2 = 938.272 \text{ MeV}$$

Hydrogen atom $m_H = 1.673533 \times 10^{-27} \text{ kg}$

$$m_H/m_u = 1.007825$$

$$m_H c^2 = 938.783 \text{ MeV}$$

Neutron $m_n = 1.67492716(13) \times 10^{-27} \text{ kg}$

$$m_n/m_u = 1.00866491578(55)$$

$$m_n c^2 = 939.565 \text{ MeV}$$

Alpha particle $m_\alpha = 6.644656 \times 10^{-27} \text{ kg}$

$$m_\alpha/m_u = 4.001506175$$

$$m_\alpha c^2 = 3727.379 \text{ MeV}$$

CONVERSION FACTORS

Fermi $1 \text{ fm} = 10^{-15} \text{ m}$

$1 \text{ eV} = 1.6022 \times 10^{-19} \text{ J}$

Million electron volts $1 \text{ MeV} = 1.602176 \times 10^{-13} \text{ J}$

$$1 \text{ MeV}/c^2 = 1.783 \times 10^{-30} \text{ kg}$$

Cross section (barn) $1 \text{ b} = 10^{-28} \text{ m}^2$

$$\text{Year} \quad 1 \text{ y} = 3.1536 \times 10^7 \text{ s}$$

USEFUL FORMULAE

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \tau \ln 2 \quad \text{where } t_{\frac{1}{2}} = \text{half life, } \lambda = \text{decay constant and } \tau = \text{mean life.}$$

Energy width of a state of lifetime τ :

$$\Gamma = 6.58212 \times 10^{-22} / \tau(\text{s}) \text{ MeV}$$

Non-relativistic speed of mass m with energy E :

$$v = 1.389 \times 10^7 [(E(\text{MeV}) / m(u))^{1/2}] \text{ ms}^{-1}$$

Non-relativistic wave number of mass m with energy E :

$$k \approx 2\pi/\lambda = 0.21874 [m(u) \times E(\text{MeV})]^{1/2} \text{ fm}^{-1}$$

Wave number for a photon of energy E :

$$k \approx 2\pi/\lambda = E / \hbar c = E (\text{MeV}) / 197.327 \text{ fm}^{-1}$$

¹ Source: 1998 CODATA Recommended Values. Uncertainties are given in parentheses.