UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

ANSWER ANY FOUR QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

VALUES OF SOME PHYSICAL CONSTANTS ARE GIVEN AT THE END OF THE PAPER

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

THIS PAPER CONTAINS FIVE QUESTIONS

Question One.

(a) (i) State what is meant by "density of states" of a system. (2 marks)

(ii). Write down the equation (in terms of energy) for the density of states in phase space for a system of classical particles.

(2 marks)

(b) (i) What is meant by degeneracy of an energy level? (3 marks)

(ii). Given that the energy $E = E_0 n^2$ where 'E_0' is a constant, 'n' is an integer and $n^2 = n_x^{\ 2} + n_y^{\ 2} + n_z^{\ 2}$ where n_x, n_y, n_z are quantum numbers 0,1,2,3 ...etc, corresponding to a quantum state. Find the degeneracy of the energy levels with energies, $11E_0$ and $26E_0$.

(5 marks)

(c) Maxwell-Boltzmann distribution function for system of classical particles is:

$$n_s = g_s \exp(\alpha + \beta\varepsilon_s)$$

(i). Use the above equation to find the multipliers α and β of a system having a total of 1000 particles distributed among two non-degenerate energy levels with energies 1 J and 2 J. Total energy of the system is 1200 J (8 marks)

(ii) Using your results in (i) above, find the most probable distribution of the particles in the two energy levels.

(3 marks)

(iii) State the Principle of Equipartition of energy. (2 marks)

<u>Question Two.</u>

(a) (i) Distinguish between an extensive and an intensive variable used in thermodynamics. Give at least one example each. (4 marks)

(ii) Write down the equation that links entropy of a system with its thermodynamic probability. (2 marks)

(iii) Two systems of identical particles 1 and 2 have entropies S_1 and S_2 and statistical weights W_1 and W_2. When the two systems are mixed together, what is:

1. The total entropy S_T (2 marks)

2. The total statistical weight W_T of the mixture? (2 marks)

(iv) Do these results agree with your equation in (ii) above? Explain. (3 marks)

$$Given: W = N! \prod_s \left\{ \frac{g_s^{n_s}}{n_s!} \right\}$$

(b) Derive an expression for the entropy 'S' of a system in terms of its partition function 'Z'.

(8 marks)

(c) Show that the partition function of a perfect classical gas can be expressed as:

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

(4 marks)

[see appendix for definite integrals]

Question Three .

(a) Write down the Bose-Einstein distribution function for a system of bosons and show under what conditions it can approximate to the classical Maxwell- Boltzmann distribution function.

(6 marks)

$$\alpha = \ln\left[\frac{Nh^3}{(2\pi mkT)^{3/2}V}\right]$$

(b) (i) State what each symbol represents in the Bose - Einstein condensation equation:

$$\frac{N'}{N} = \left(\frac{T}{T_B}\right)^{3/2}.$$

(2 marks)

(ii) Find the relationship between the number of particles N_0 in the ground state and the temperature.

(4 marks)

(iii) Draw a sketch to show how N_0 varies with temperature.

(3 marks)

(c) In a Bose - Einstein condensation experiment, 10^7 rubidium atoms (mass = 1.425×10^{-22} g) were cooled down to a temperature of 200 nK. The atoms were confined to a volume of 10^{-15} m^3 .

(i) Calculate the condensation temperature. (6 marks)

(ii) Find how many atoms were in the ground state at 200 nK.

$$\text{given } T_B = \frac{h^2}{2\pi mk}\left(\frac{N}{2.612V}\right)^{2/3}$$

(4 marks)

Question Four.

(a) Given that a one-dimensional harmonic oscillator has discrete energy given by $\varepsilon = \left(n + \frac{1}{2}\right)h\nu$, where symbols have their usual meanings, obtain an expression for its mean energy. (9 marks)

Given: mean energy $= kT^2\left(\frac{\partial \ln Z}{\partial T}\right)_V$

(b) Assume that a solid has N atoms each having three mutually independent vibrations. Using your results in question (a) above, obtain an expression to show how the specific heat capacity of the solid varies with temperature T. (8 marks)

(c) Show that at high temperatures specific heat capacity of the solid is equal to the classical value 3Nk. (4 marks)

(d) Experimentally it is observed that at low temperatures specific heat of solids varies proportional to T^3. Does the above theory agree with this? Comment. (4 marks)

Question Five.

(a) The Fermi function of system is given as: $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/kT} + 1}$ where symbols have their usual meanings. Obtain its values at absolute zero temperature, for the cases $\epsilon > \epsilon_F$ and $\epsilon < \epsilon_F$.

What is the physical meaning of the these results? (6 marks)

(b) The Fermi level of a solid 8.6 eV. Find the probability of occupation of electron:

(i) in an energy level 0.1 eV above the Fermi level at 300K and at 400K

(ii) in an energy level 1.0 eV above the Fermi level at 300K and at 400K

Comment on the results. (6 marks)

(c) Derive an expression for the paramagnetic susceptibility of a metal to show that it is independent of temperature. Draw diagrams where necessary

[Neglect the response to the applied field due to the orbital motion of the electrons. Assume energy $\mu_B B \ll \epsilon_F$ and ϵ_F is a constant for the material]

(13 marks)

Given: the density of states per unit volume: $g(\varepsilon)d\varepsilon = \dfrac{4\pi}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon.$

APPENDIX.

Physical Constants.

quantity	*symbol*	*value*
Speed of light	c	3.0×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	q	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Avogadro's number	N	6.02×10^{23}

Appendix

Definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty x^{1/2}e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x-1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

$$\int_0^\infty e^{-ax^2}x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty e^{-ax^2}x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x-1} = \frac{\pi^4}{15}$$