UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE OF THE PAPER: COMPUTATIONAL PHYSICS-II

COURSE NUMBER : P482

TIME ALLOWED : *SECTION A: ONE HOUR.*
SECTION B: TWO HOURS.

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS PAPER:

SECTION A **: THIS IS A WRITTEN PART ON YOUR ANSWER BOOK. CARRIES A TOTAL OF 30 MARKS.**

***SECTION B*: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF 70 MARKS.**

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **BOTH** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS FOUR PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section)

Q.1. (a) Write short notes on

(i) Random numbers generated on a computer. [5]

(ii) Method of Monte Carlo Integration. [5]

(b) Give a pseudo code to calculate the integral of the form [5]

$$Integral = \int_a^b dx \int_c^d dy\, f(x,y)$$

using Monte-Carlo method.

Q.2. Under the Block-Gruneissen approximation for the resistance in a mono-valent metal, integrals of the following form need to be calculated;

$$\text{BGintegral} = \int_0^2 \frac{x^5}{(e^x - 1)(1 - e^{-x})}\, dx$$

To avoid the singularity at x=0, assume lower limit of integration to be 0.001.

Write a **psuedo-code** to calculate the integral using Simpson Rule for t=2 to a precision of 0.001 [15]

Q.3. Write a program using Maple syntax to simulate following noisy signal of 256 data points *p(i)* with time steps of $h = 4\pi/256$ seconds using [15]

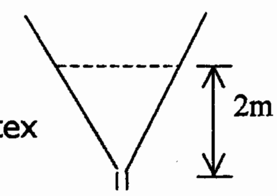
$$p(i) = \cos(t_i) + 4.0 * RAN(x_i)$$

$$p(i + j\Delta) = \cos(t_i + j\Delta) + 4 * RAN(x_{i+j+1})$$

for $t_i = i\,h$ where $i = 1,2,3,....n$, $j = 1,2,3,..n-1$ *and* $n=256$.
Here $RAN(x_i)$ *is* i^{th} *random number in the range (0,1). For each data point You may select any* Δ *such that* $t_i + j\,\Delta$ *should always correspond to one of the 256 data points in the interval* $(0, 4\pi)$ *.*

SECTION B
(Practical Section)

Q.4. Water flows from an inverted conical tank with the circular orifice (as shown in the figure) at the rate



$$\frac{dx}{dt} = f(x) \qquad \text{where} \quad f(x) = -.0475\,x^{-3/2}$$

where x is the height of the liquid level from the vertex of the cone and time **t is in units of minutes**.
Initial water level is at 2m.
Compute the water level after 10 minutes

(i) Using **dsolve** command of Maple, determine the exact solution. [5]

(ii) Write a procedure using Euler method to solve the given differential equation and use it to find x(t) in the interval 0<t<10minutes . [25]

(iii) Plot the solution of (i) and (ii) on one graph. Comment. [5]

Q.5. Consider a two dimensional lattice. The distance between successive lattice points is of unit length. [35]

Assume that all the lattice points at ***(x ,y=20)*** and ***(x ,y=-20)*** for all x have **reflecting property** and all the lattice points at ***(x=20, y)*** and ***(x=-20, y)*** have **a sink (a point of no return).** That is if a walker reaches y=20 the next step is towards south or if the walker reaches y=-20, the next step is towards north. Here we have assumed y-direction to be north-south. On the other hand, if a walker reaches ***x=20*** or ***x=-20*** for any ***y***, a point of no return is encountered (that is the walk comes to an end).

Write a program and execute it to find the **number of steps** a walker would take before encountering the **point of no return** in **x-direction**. Assume that the random walk starts from (x=0,y=0).

Note: (i) The number of steps required to reach the point of no return can be more than the number of lattice points. (ii) Use the uniform random number generator available in Maple.

@@@@END OF EXAMINATION@@@@

APPENDIX

1. Solution of First Order Differential Equation with initial Conditions:

The equation is of the form $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = \alpha$.

(i) Euler's Method:

$$y_{i+1} = y_i + h f(x_i, y_i) \qquad \text{where } h = x_{i+1} - x_i$$

(ii) Fourth Order RK-Method:

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$k_1 = h f(x_i, y_i)$
$k_2 = h f(x_i + 0.5h, y_i + 0.5k_1)$
$k_3 = h f(x_i + 0.5h, y_i + 0.5k_2)$
$k_4 = h f(x_i + h, y_i + k_3)$ *and* $h = x_{i+1} - x_i$

2. Numerical Integration:

Simpson Rule:

$$\int_a^b f(x)dx = \frac{h}{3}[f(a) + f(b) + 4(f_1 + f_3 + f_5 + \ldots\ldots + f_{n-1}) + 2(f_2 + f_4 + f_6 + \ldots + f_{n-2})]$$

where $f_0 = f(a)$, $f_n = f(b)$, $f_1 = f(a+h)$, $f_2 = f(a+2h)$......*etc. and* n=*even integer.*