UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006/07

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR

QUESTION 1

(a) Two balls A and B are connected by a rigid rod of length L. The balls slide along perpendicular guide rails as shown in Figure 1. A slides to the left with constant velocity υ. Find the velocity of B in terms of υ when the angle α becomes 60°. **(7 marks)**

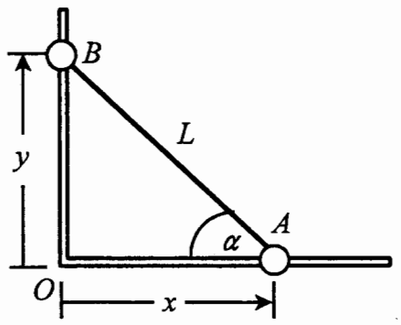


Figure 1.

(b) An elevator ascends from the ground with uniform speed. At time T_1 a boy drops a marble through the floor of the elevator. The marble falls with gravitational acceleration g and hits the ground T_2 seconds later. Find the height of the elevator at time T_1 in terms of T_1, T_2 and g. **(7 marks)**

(c) A tyre rolls in a straight line without slipping (see Figure 2). Its centre moves with constant speed V. A small stone lodged in the tread at the circumference of the tyre touches the road at $t = 0$. Find the stone's position, velocity and acceleration as functions of time in Cartesian coordinates. **(6 marks)**

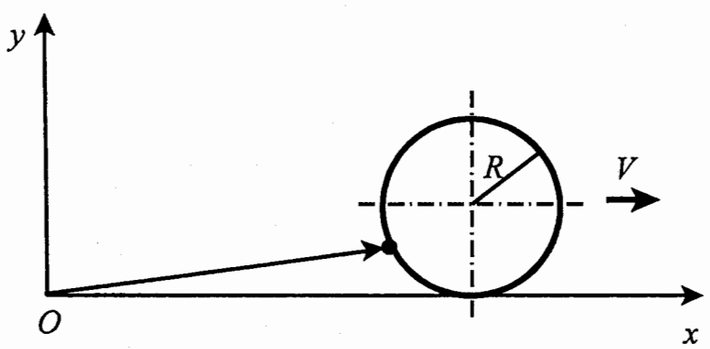


Figure 2.

(d) Derive the kinematic equation:

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0).$$ **(5 marks)**

QUESTION 2

(a) Two blocks are in contact on a horizontal frictionless table. A horizontal force is applied to one of the blocks as shown in Figure 3. If $m_1 = 4$ kg, $m_2 = 3$ kg, and $F = 8$ N, find the force of contact between the two blocks. **(4 marks)**

(b) A and B are fixed points on a vertical pole at a distance 6 m apart. A particle of mass m is attached to A and B by inextensible strings of negligible mass, each of length $l = 5$ m. The path of the particle is a horizontal circle at constant speed ω_0. See Figure 4.

(i) Draw a clear resolved force diagram for the mass, from which useful equations can be deduced. **(4 marks)**

(ii) Find the tensions T_1 and T_2 in the strings in terms of m and g if the mass m moves in a horizontal circle with angular velocity $\omega_0 = (4g)^{1/2}$. First find the general solutions and substitute at the end for ω_0. **(9 marks)**

(c) In the system shown in Figure 5, find the acceleration of each mass in terms of m_1, m_2 and g. Assume that all surfaces are frictionless and that the masses of the pulleys are negligible. **(8 marks)**

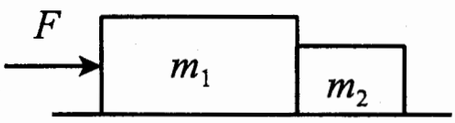


Figure 3.

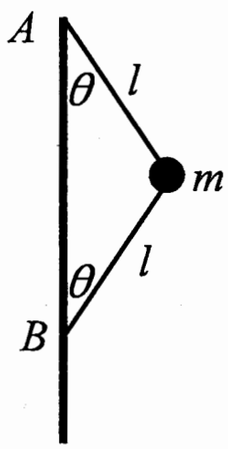


Figure 4.

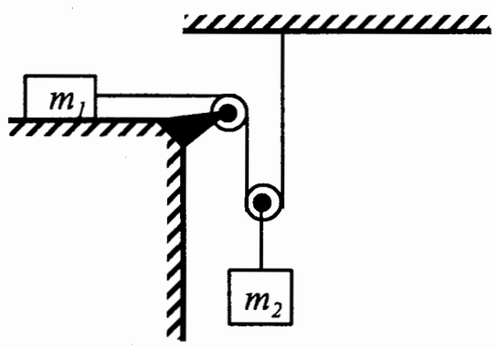
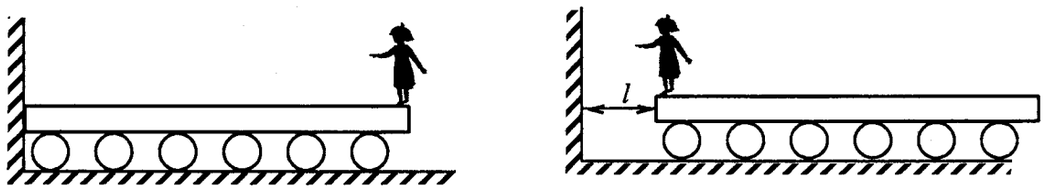


Figure 5.

QUESTION 3

(a) A child of mass $m = 40$ kg stands at one end of a plank of mass $M = 70$ kg and of length $L = 4$ m as shown in Figure 6. The plank is placed on top of light ball bearings (to make the friction with the floor negligible) with the other end on the opposite side of the child against a wall. The child starts to walk towards the other end of the plank. How close will the child get to the wall? That is find the distance l. **(8 marks)**



(a) **(b)**

Figure 6.

(b) An empty rail car of mass M_0 starts from rest under an applied force F. At the same time sand starts to run into the car at a steady rate b (in kg/s) from a hopper at rest along the track.

(i) Find the velocity of the rail car as a function of time. **(10 marks)**

(ii) What is the terminal velocity reached by the rail car? **(2 marks)**

(c) Water shoots out of a fire hydrant having a nozzle radius R with speed V_0. What is the reaction force on the hydrant. **(5 marks)**

QUESTION 4

(a) A body of mass m is launched vertically upward from a radius r_0 with an initial velocity υ_0. It is acted upon by an inverse square force, $F = -Am/r^2$, where r is the displacement from the launching point and A is a constant. What is the maximum distance reached by the body in terms of A, r_0 and υ_0?

(6 marks)

(b) Two paths are provided for a particle to move from (0,0) to (2R,0) under the force $F = A(xy\hat{i} + y^2\hat{j})$. The first path is straight line along the x-axis and the second is along the circumference of a semicircle of radius R as shown in Figure 7.

(i) What is the work done along the straight line path? **(2 marks)**

(ii) What is the work done along the curved path? **(8 marks)**

(iii) Comment on the nature of the force F. **(2 marks)**

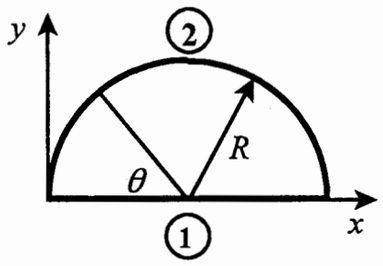


Figure 7.

(c) Consider a potential like the one shown in Figure 8. Show that a particle at the bottom of the well will oscillate harmonically when disturbed slightly from the equilibrium point r_0.

(7 marks)

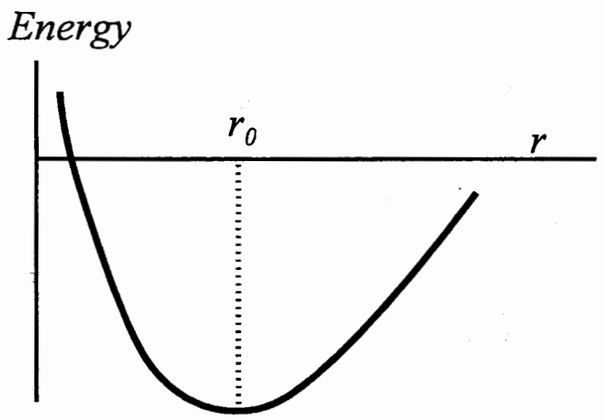


Figure 8.

QUESTION 5

(a) Find the moment of inertia of a hollow uniform disk of thickness t, inner radius R_1, outer radius R_2, and mass M in terms of R_1, R_2 and M. **(7 marks)**

(b) A very light string is wound around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its free end attached to a rigid point above (see Figure 9). Find the tension in the string in terms of m and g. **(6 marks)**

(c) A wooden block of mass M resting on a frictionless horizontal surface is attached to a rigid rod of negligible mass and length l (see Figure 10). The rod is pivoted at the other end. A bullet of mass m traveling parallel to the horizontal and perpendicular to the rod with speed υ_0 hits and embeds itself in the block.

(i) What is the angular momentum of the bullet-block system in terms of m, υ_0, and l.? **(2 marks)**

(ii) Use the angular momentum method to find the final velocity just after the collision. **(3 marks)**

(d) A wheel is attached to a fixed shaft, and the system is free to rotate without friction (see Figure 11). To measure the moment of inertia of the wheel-shaft system, a tape of negligible mass wrapped around the shaft is pulled with a known constant force F. When a length L of the tape has unwound, the system is rotating with angular speed ω_f. Find the moment of inertia of the system I_0 in terms of L, F and ω_f. **(7 marks)**

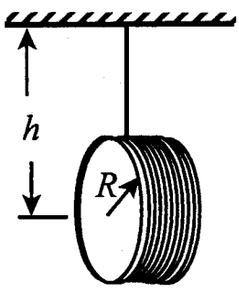


Figure 9.

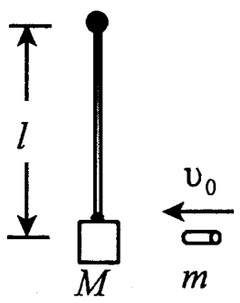


Figure 10.

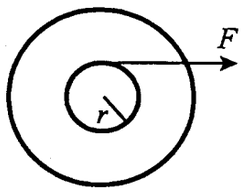


Figure 11.