UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS AND ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2006/07

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Derive the kinematic equation

$$\upsilon^2 = \upsilon_0^{\ 2} + 2a(x - x_0).$$ **(5 marks)**

(b) Show that $\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$ in plain polar coordinates. **(5 marks)**

(c) Write down the vector $\vec{r}$ to a point P in spherical coordinates, and show how it is obtained using a clear diagram. **(5 marks)**

(d) Find the volume of a quarter of a hollow sphere of inner radius R_1 and outer radius R_2. Note: the volume element is given by $d\tau = r^2 dr \sin\theta d\theta d\phi$. **(5 marks)**

(e) Write down the acceleration in plain polar coordinates and explain the meaning of each term. **(5 marks)**

QUESTION 2

(a) In the system shown in Figure 1, find the acceleration of each mass in terms of m_1, m_2 and g. Assume that all surfaces are frictionless and that the masses of the pulleys are negligible. (Note: $a_2 = 2a_1$). **(7 marks)**

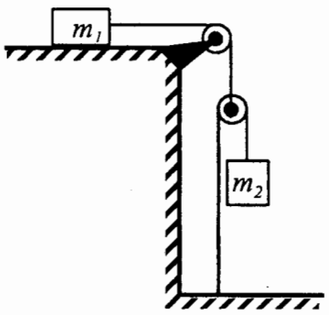


Figure 1.

(b) A mass m is attached by two strings to a vertical rod, as shown in Figure 2. The entire system rotates with constant angular velocity ω about the axis of the rod. Assume that ω is large enough to keep both strings taut, making the lower string of to be horizontal and the upper string of length l to make an angle θ with the rod.

(i) Make a complete force diagram for the mass m. **(3 marks)**
(ii) Find the tension in each string in terms of m, g, R, and θ. **(6 marks)**
(iii) What is the minimum angular velocity ω_{min} for the lower string to be just taut. **(4 marks)**

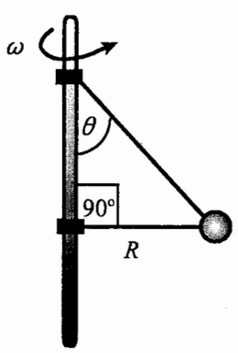


Figure 2.

(c) A flat-bed truck carries a box of mass m and moves around a bend of radius R. Make a force diagram for the box and write down the equations for the box to remain on the truck without sliding off. **(5 marks)**

QUESTION 3

(a) The density of a thin rod of length varies with the distance x from one end as $\lambda = \lambda_0 x^2/l^2$, where λ_0 is a constant. Find the position of the centre of mass of the rod. **(5 marks)**

(b) An inverted garbage can of weight W is suspended in air by water shooting out vertically from a pipe with an initial velocity υ_0 at a rate k (kg/s). What is the maximum height reached by the garbage can. **(6 marks)**

(c) A rain drop of mass falls from rest under gravity. Its motion is resisted by air resistance whose magnitude is $F_R = k\upsilon$, where k is a constant.

(i) Make a force diagram and find an expression for the acceleration of the rain drop. **(3 marks)**

(ii) Find the velocity of the rain drop as a function of time. **(9 marks)**

(iii) What is the terminal velocity of the rain drop? **(2 marks)**

QUESTION 4

(a) A body is projected in the positive r direction. It is acted upon by an inverse square force, $F = -A/r^2$, where r is the displacement from the launching point and A is a constant. What is the potential energy of the body as a function of r? **(6 marks)**

(b) Two paths are provided for a particle to move from (0,0) to (2R,0) under the force $F = A(x^2\hat{i} + xy\hat{j})$. The first path is straight line along the x-axis and the second is along the circumference of a semicircle of radius R as shown in Figure 3.

(i) What is the work done along the straight line path? **(2 marks)**
(ii) What is the work done along the curved path? **(8 marks)**
(iii) Comment on the nature of the force F. **(2 marks)**

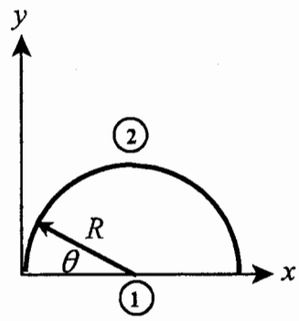


Figure 3.

(c) Show that a system with potential energy U and kinetic energy K of the following forms, respectively:

$$U = \frac{1}{2}Aq^2 + C,$$

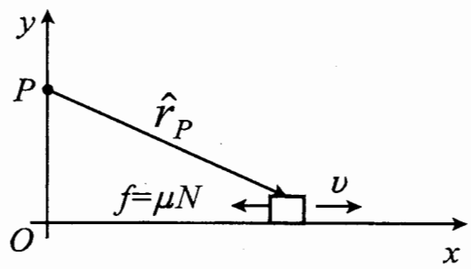
$$K = \frac{1}{2}B\dot{q}^2,$$

oscillates harmonically, where A, B, and C are constants with appropriate units, and q is a variable appropriate to the particular problem. **(7 marks)**

QUESTION 5

(a) A block slides in the positive x direction and is slowed down by a frictional force $f = \mu N$, where N is the normal force and μ is the coefficient of kinetic friction (See Figure 4). Find the angular momentum and the torque about an axis through point *P* at $y = l$. **(8 marks)**

Figure 4.



(b) Find the moment of inertia of a stick of length *L* about an axis through its centre. Thelength density of the stick varies as $\lambda = \frac{\lambda_0 x}{L}$, where λ_0 is a constant. **(6 marks)**

(c) The physical pendulum can be made of a body of any shape which is pivoted a distance *l* from its centre of mass.

(i) Use derivation to show that it oscillates harmonically. **(8 marks)**

(ii) Obtain an equation for its angular velocity in terms of the gravitational acceleration *g*, the distance from the pivot *l*, and the radius of gyration *k*. **(3 marks)**