UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE
DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006/2007

TITLE OF THE PAPER: **COMPUTATIONAL PHYSICS -I**

COURSE NUMBER : **P262**

TIME ALLOWED : **THREE HOURS**

SECTION A: ONE HOUR.
SECTION B: TWO HOURS.

INSTRUCTIONS:

SECTION A : THIS IS THE **WRITTEN PART** TO BE COMPLETED IN YOUR ANSWER BOOK. CARRIES A TOTAL OF 30 MARKS.

SECTION B: THIS IS THE **PRACTICAL PART** WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF 70 MARKS.

- ANSWER **ANY TWO** QUESTIONS FROM **SECTION A .**
- **ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS THREE PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section) **Time: One Hour**

Q.1. (A) Explain the differences in the following:

(i) f : = x->x^2+cos(x) and g : = x^2+cos(x) . [1]

(ii) commands **solve** and **fsolve.** [1]

(B) Translate the following in Maple. [2]

(i) a $b^{1/2}$ e^{-x} sin(x) (ii) $x^{a/c}$ +(y^b / $z^{1/2}$ +1) tan(p)

(C) Use Maple command ***sum*** to calculate [2]

$$1+\frac{1}{5^3}+\frac{1}{9^5}+\frac{1}{13^7}+\cdots\cdots$$

for the first 20 terms of the series.

(D) The velocity v_c necessary to maintain the circular orbit of a satellite at a distance of h (m) above the surface of the Earth is given approximately by

$$v_c=\frac{7749\sqrt{R}}{\sqrt{R+h}}$$

where R = radius of Earth = 6.2712 x 10^6 m .

Write a program to calculate v_c for h_i = 3000+ *i* Δh for *i*=1,2,.....10 given that Δh = 1000 m . [9]

Q.2. (A) Current standard in arithmetic calculations in PC's is based on 32 bit computer word in single precision, 64 bit computer word in double precision and this restricts the precision allowed for a calculation.

(i) Give an estimate of the floating point precision in single and double precision respectively. [2]

(ii) In Maple using the same computer one can have higher precision of calculation. Explain the mechanism used in Maple to achieve this. [3]

(B) Bonnet recursion formula for Legendre Polynomial is given by [10]

$$(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x) \qquad \text{for } n = 1,2,3.....$$

Given that $P_0(x)=1$, $P_1(x)=x$, write a pseudo-code to calculate $P_n(x)$ for any $n \geq 2$.

Q.3. (i)Write a program to calculate a sequence of $S(t)=ut+\frac{1}{2}at^2$ for $0 \leq t \leq 2$ with an increment $\Delta t=0.05$. [5]

Assume parameters u = *1.5* and *a=9.8* .

(ii) Plot *S(t)* vs *t*. [2]

(iii) Convert the program of (i) above into procedure with the name **displ** which can be used for any values of parameters. [5]

(iv) Use the procedure of (iii) to calculate s(t) vs t for a=9.8 and u=1.5,2.0,and 2.5. [3]

SECTION B
(Practical Section) **Time: Two Hours.**

Q.4. (a) Plot the function $f(x) = cos^2(x) - x^3 + x$ for x = -3 to 2. [2]
How many real roots are there? [1]
Solve the equation $f(x)=0$ to determine all its real roots. [7]

(b) $I(\lambda)$, the intensity distribution of radiation from a blackbody at a temperature T , may be represented by a formula [15]

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^3} \frac{1}{exp(hc/\lambda kT) - 1}$$

where λ wavelength in metres, T is the temperature in Kelvins, c= 3 x 10^8 ms^{-1}, h= 6.63 x 10^{-34} Js and k= 1.38 x 10^{-23} JK^{-1}.

Write a program to calculate a value of $I(\lambda)$ at temperature T=2200 K over the visible wavelength range 0.4 μm to 0.7μm for 100 different values of λ at equal intervals.
Plot the sequence $I(\lambda)$ vs λ

(c) Write a program
(i) to generate data points (x_i, t_i) for $0 \le t \le 2$ in steps $\Delta t = 0.1$ where $x_i = 1.2 + 8\,t_i^2$. [5]
(ii) use the above data to calculate [5]
$\sum(x_i^2 - x_i t_i)$

Q.5. A ball is thrown straight upward from the top of a cliff with an initial velocity of 25 ms^{-1} . In addition to the action of gravity on the ball, a viscosity-limited drag force opposes the motion of the ball. This drag force is proportional to the speed of the ball and so the net acceleration is

$$a(t) = -g - \alpha v(t)$$

where a(t) and v(t) are the acceleration and velocity of the object respectively at time t. α is a constant related to air resistance. Take α=1.1 s^{-1}.
Using Maple commands,
(i) Solve the differential equation to obtain analytical solution for v(t), and determine the time taken for the object to start falling. [15]
(ii) Convert the equation into two first order differential equations involving position x(t) and the velocity $v(t) = \frac{dx(t)}{dt}$. [2]
Solve the two equations numerically using Maple commands. [8]
Plot x(t) and v(t) vs t for t=0..5 s , on the same graph. [6]
Determine the position x(t) when the object starts falling using the time determined in (i). [4]

@@@END OF EXAMINATION@@@