UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006/2007

TITLE OF PAPER	:	**CLASSICAL MECHANICS**
COURSE NUMBER	:	**P320**
TIME ALLOWED	:	**THREE HOURS**
INSTRUCTIONS	:	**ANSWER ANY FOUR OUT OF FIVE QUESTIONS** **EACH QUESTION CARRIES 25 MARKS** **MARKS FOR DIFFERENT SECTIONS OF EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN**

THIS PAPER HAS 10 PAGES, INCLUDING THIS PAGE.

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PHYSICAL CONSTANTS AND UNITS

Acceleration due to gravity	g	$9.81\ m\ s^{-2}$
Gravitational constant	G	$6.673 \times 10^{-11}\ N\ m^2\ kg^{-2}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ mol^{-1}$
	(Note: 1 mole = 1 *gram* molecular-weight)	
Ice point	T_{ice}	273.15 K
Gas constant	R	$8.314\ J\ K^{-1}\ mol^{-1}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ J\ K^{-1} = 0.862 \times 10^{-4}\ eV\ K^{-1}$
Stefan constant	σ	$5.670 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$
Rydberg constant	R_∞ $R_\infty hc$	$1.097 \times 10^7\ m^{-1}$ 13.606 eV
Planck constant	h	$6.626 \times 10^{-34}\ J\ s = 4.136 \times 10^{-15}\ eV\ s$
$h/2\pi$	$\hbar$	$1.055 \times 10^{-34}\ J\ s = 6.582 \times 10^{-16}\ eV\ s$
Speed of light *in vacuo*	c	$2.998 \times 10^8\ m\ s^{-1}$
	$\hbar c$	197.3 MeV fm
Charge of proton	e	$1.602 \times 10^{-19}\ C$
Mass of electron	m_e	$9.109 \times 10^{-31}\ kg$
Rest energy of electron		0.511 MeV
Mass of proton	m_p	$1.673 \times 10^{-27}\ kg$
Rest energy of proton		938.3 MeV
One atomic mass unit	u	$1.66 \times 10^{-27}\ kg$
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	$8.854 \times 10^{-12}\ F\ m^{-1}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ H\ m^{-1}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ A\ m^2\ (J\ T^{-1})$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ A\ m^2\ (J\ T^{-1})$
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_c = h/mc$	$2.426 \times 10^{-12}\ m$
Bohr radius	a_0	$5.2918 \times 10^{-11}\ m$
angstrom	Å	$10^{-10}\ m$
torr (mm Hg, 0°C)	torr	$133.32\ Pa\ (N\ m^{-2})$
barn	b	$10^{-28}\ m^2$

Question one

Consider a particle moving under the influence of a central conservative force. There are two conservation laws, one for energy and one for angular momentum, namely

$$\frac{1}{2}m\dot{\mathbf{r}}^2 + U(r) = \text{constant} \qquad\qquad m\mathbf{r}.\dot{\mathbf{r}} = \mathbf{J} = \text{constant.}$$

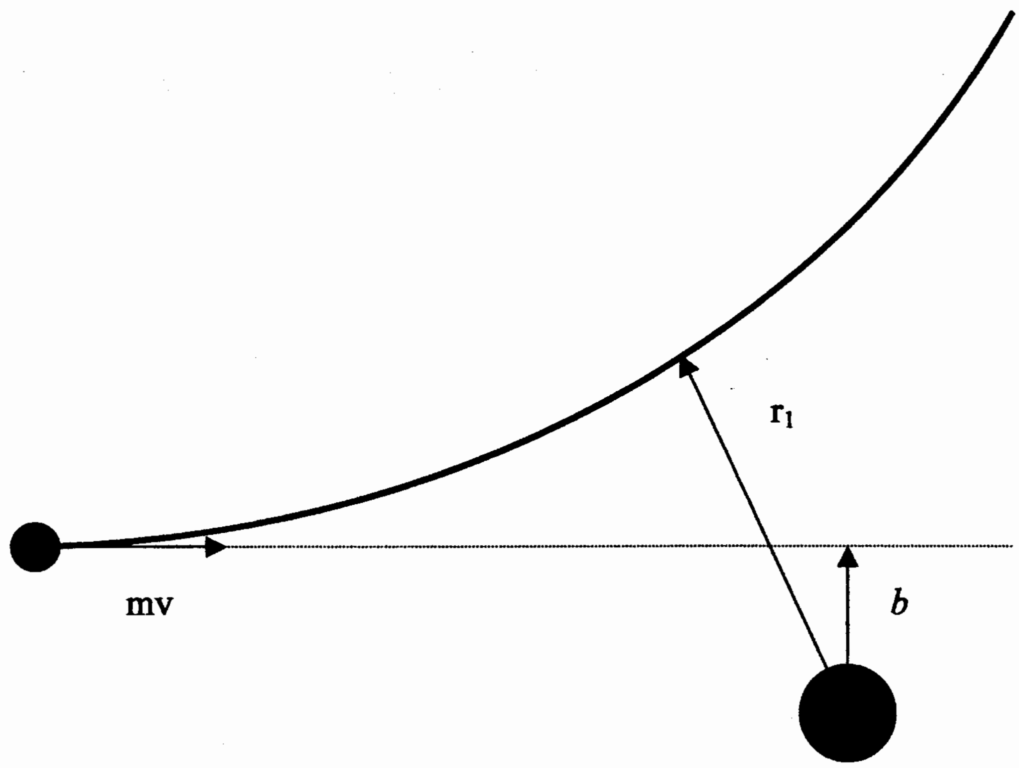
The second of these laws implies that the motion is confined to a plane. Explain why this is the case. [5]

Recast the equations in terms of polar co-ordinates, r and θ in this plane. Obtain the so-called *radial energy equation* by eliminating the time-dependence of the angular co-ordinate θ. What is the origin of the term in the *effective potential energy* which is proportional to J^2? [5]

Rutherford designed an experiment, carried out by Geiger and Marsden at the Cavendish Laboratory in 1911, which involved the scattering of alpha particles by the nuclei of gold atoms. What is the nature of the interaction between the alpha particles and the gold nuclei, and why can the interactions between the electrons surrounding the gold atoms and the alpha particles be neglected? Describe the outcome of the experiment. [5]

If there were no interaction between the alpha particles and the gold nuclei, then a particular alpha particle would pass a distance b from a certain gold nucleus, where b is the impact parameter. See the diagram below.

trajectory of the alpha particle is depicted as the heavy line in the diagram, the alpha particle as the smaller black circle, and the gold nucleus as the larger black circle.



Derive an expression for the closest approach of an alpha particle and gold nucleus - r_1 in the diagram - in terms of parameters involving the interaction and the impact parameter, *b*. [10]

Question three

Consider a rigid body, free to rotate about a fixed axis, taken to be the z-axis. The origin of the co-ordinate system is chosen to lie on this axis, so that the z co-ordinate of the centre of mass of the body is 0. In cylindrical polar co-ordinates the z and ρ co-ordinates of every point in the body are fixed, while φ varies as $\dot{\varphi} = \omega$, the angular velocity of the body.

The angular momentum of the body is usually written as $J_z = I\omega$. What is I?

[5]

If the density of the rigid body is $\sigma(\mathbf{r})$, write down an expression for I in terms of $\sigma(\mathbf{r})$ and ρ. [5]

What is the kinetic energy of the rotating body? [5]

Consider a rigid body pivoted about a horizontal axis and moving under gravity – see the diagram below. The z-axis is the axis of rotation, which is perpendicular to the plane of the paper, and gravity acts in the downwards x-direction. **R** gives the distance of the centre of mass from the axis of rotation. The force acts at the centre of mass, and has the component (Mg, 0, 0).

Show that the period for small oscillations of the rigid body is $2\pi\sqrt{(I/MgR)}$.

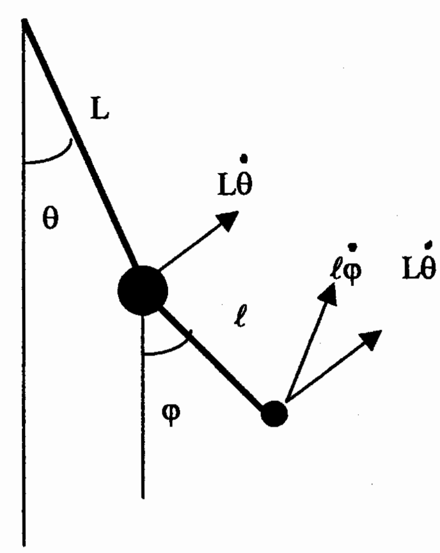
[10]

Question four

Any sensible co-ordinate system, used to describe the positions and velocities of a set of particles, uses orthogonal co-ordinates. What does this mean? [2]

Show that the three unit vectors used in the Cartesian, cylindrical and spherical polar systems are orthogonal. [5]

Consider the motion of the bobs in the double pendulum system in the figure below. The larger bob has mass, M, while the smaller bob has mass m. The motion of both bobs is restricted to lie in the plane of the paper.



Show that the kinetic energy of the system is

$$T = \frac{1}{2}ML^2\dot{\theta}^2 + \frac{1}{2}m\left[L^2\dot{\theta}^2 + \ell^2\dot{\varphi}^2 + 2L\dot{\theta}\ell\dot{\varphi}\cos(\varphi-\theta)\right].$$ [10]

Explain why the co-ordinates φ and θ are not orthogonal. [2]

For small values of θ and φ, $\cos(\varphi - \theta)$ may be set equal to unity. Introduce two orthogonal parameters that may be used to describe the motion of the two bobs, and write down an expression for the kinetic energy in terms of these two parameters. [6]

Question five

Consider a system of 2 particles that interact with a force **F**, and experience the same uniform gravitational field. Explain why the equations of motion for the two particles can be written as:

$$m_1 \ddot{\mathbf{r}}_1 = m_1\mathbf{g} + \mathbf{F}$$

$$m_2 \ddot{\mathbf{r}}_2 = m_2\mathbf{g} - \mathbf{F}$$ [6]

To describe the motion of these two particles it is convenient to introduce parameters that describe the position of their centre of mass, **R**, their relative positions, **r**, and their reduced mass, μ. These quantities are defined as follows:

$$\mathbf{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \qquad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}.$$

Why is μ called the reduced mass? [2]

Show that

$M\ddot{\mathbf{R}} = M\mathbf{g}$ and $\mu\ddot{\mathbf{r}} = \mathbf{F}$ where $M = m_1 + m_2$. [5]

Also show that the total angular momentum, **J**, and total kinetic energy, T, of the two particles may be written as

$$\mathbf{J} = M\mathbf{R}\times\dot{\mathbf{R}} + \mu\mathbf{r}\times\dot{\mathbf{r}}$$

$$T = \frac{1}{2}M\dot{\mathbf{R}}^2 + \frac{1}{2}\mu\dot{\mathbf{r}}^2 .$$ [6]

The two particles collide *elastically*. What can you deduce about the total kinetic energy of the particles before and after this collision? In the centre of mass frame of reference, what is the total momentum of the two particles? In the same frame of reference, if one of the particles is scattered through an angle of 30°, what angle has the other particle been scattered through?