UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2007

TITLE OF PAPER	:	ELECTROMAGNETIC THEORY
COURSE NUMBER	:	P331
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS OF EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE

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Question 1

Using Gauss's theorem and the fact that the electric field, $\boldsymbol{E}$, is the negative of the spatial derivative of the scalar potential, V, derive Poisson's and Laplace's equations. [5]

One method of obtaining an algebraic solution to these equations involves a mathematical technique known as *separation of the variables*. Indicate how a partial differential equation involving all three spatial variables can be transformed into three simple, ordinary differential equations in x, y and z. [5]

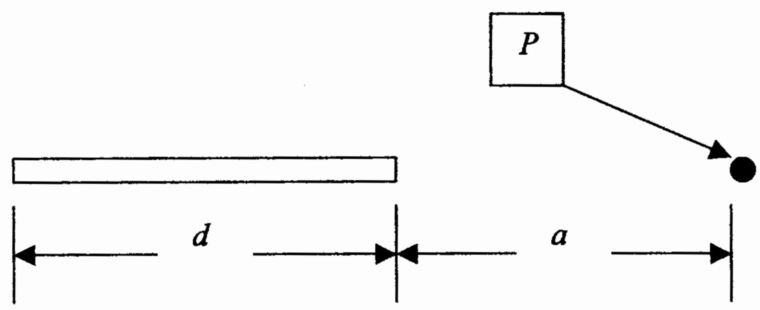
Use Laplace's equation to find the capacitance of a parallel plate capacitor. Neglect edge effects. Explain carefully each step in your derivation. [15]

Question 2

Write down expressions for the Biot-Savart law and Ampère's circuital law that relate the magnetic flux density $\boldsymbol{B}$ to a conventional electric current I. Are these relationships valid if the current is changing with time? [5]

A planar, circular loop of wire with radius r carries a conventional current of I. Use the Biot-Savart law to derive an expression for the magnetic flux density $\boldsymbol{B}$ at the centre of the loop. [5]

A very long, straight, thin copper ribbon of width d carries a steady current of I ampères. Derive an expression for the flux density $\boldsymbol{B}$ at a point P, shown as the black dot in the diagram, which lies in the plane of the ribbon at a distance a from its nearer edge. In the diagram below, the length of the copper ribbon is perpendicular to the plane of the paper.



If $I = 20$ ampères and $a = d = 0.1\ m$, what is the magnitude of $\boldsymbol{B}$ at the point P? [15]

[Hint: divide the copper slab into infinitesimal widths dx and assume that the current flowing within that portion of the ribbon is equal to Idx/d and then perform an integral of dx over the length d and evaluate this from a to $(a + d)$].

Question 3

What is the relationship between Poynting's vector, ***S***, the energy density of an electromagnetic wave, *U*, and the velocity of the wave? [2]

In the first part of this question, should the group or phase velocity of the wave be used?
What are the dimensions of ***S***? [3]

The spatial and time dependence of the electric and magnetic fields of a wave travelling in free space may be written as

$$\mathbf{E} = \{real\} \exp j(\omega t - kz) \hat{\mathbf{x}}$$

$$\mathbf{H} = \{real\} \exp j(\omega t - kz) \hat{\mathbf{y}}$$

Describe the meaning of the terms in the expressions for **E** and **H**.
In which direction is the wave travelling?
Is the wave transverse or longitudinal? [4]

Derive an expression for the time-averaged value of Poynting's vector for an electromagnetic wave using the complex notation for ***E*** and ***H***, given above, to represent the electromagnetic wave. [4]

A straight, conducting wire of radius ***r***, conductivity σ, is parallel to the z - direction and carries a steady current *I*. Determine the total power entering the wire per metre. Express this in terms of the current *I*, and the resistance of the wire *R*.
In which direction does the energy flow? [12]

Question 4

Starting from Maxwell's equations, show that the wavenumber k for a plane electromagnetic wave propagating in a medium with relative permittivity ε_r, relative permeabibility μ_r, and conductivity σ is given by

$$k^2 = \varepsilon_r \mu_r k_0^2 \left[1 - \frac{j\sigma}{\omega\varepsilon} \right]$$

where k_0 is the wavenumber in free space, ω the angular frequency, $\varepsilon = \varepsilon_r\varepsilon_0$ and $j = \sqrt{-1}$.

[$\nabla \times \nabla \times V = -\nabla^2 V + \nabla(\nabla . V)$, where V is a vector] [10]

Most microwave ovens operate at 2.45 GHz. At these frequencies, beef has a relative permittivity of about 50 and a conductivity of about 2 $(\Omega.\text{m})^{-1}$. At this frequency beef could be treated as a poor conductor. Is this a reasonable approximation? [3]

Treating beef as a poor conductor at 2.45 GHz

(a) estimate the penetration depth of the microwaves into the joint of beef [4]
(b) estimate what fraction of the power of the microwaves, once inside the joint of beef, is absorbed by the meat. [4]
(c) does cooking beef in a microwave oven offer any advantages over conventional (infrared) cooking? [4]

Question 5

Derive the boundary conditions for the tangential and normal components of ***B*** and ***H*** at a planar interface between a magnetic material and air. [8]

The relative permeability of a particular magnetic material has a magnitude of 7,000. What are the dimensions of the relative permeability? [1]

Consider a situation in which ***B*** in this magnetic material is normal to the planar interface between the magnetic material and air. Evaluate the angle that ***B*,** in air, makes with the normal to the surface. [8]

Now consider the situation in which ***B*** in the same magnetic material is nearly tangential to the surface, making an angle of 85° to the normal to the interface. What angle does ***B*** make with the normal to the surface in air?

[8]