UNIVERSITY OF SWAZILAND.

FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2006/2007.

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One.

(a) (i) Define ***unit cell*** of a crystal. (2 marks)

(ii) Distinguish between a primitive unit cell and a conventional unit cell. (2 marks)

(iii) Draw the Wigner - Seitz cell of a two-dimensional direct lattice. State whether it is a primitive or a conventional cell. (2+1 marks)

(b) (i) In the diagram of a cubic unit cell, show a (200) and a $(\bar{1}00)$ plane. (4 marks)

(ii) A cubic crystal plane has intercepts 3a, 2a and 1a along x, y, and z axes where a is the lattice constant.
Find the Miller indices of this plane.
Calculate the separation between two such planes if the lattice constant 1 Å. (2+2 marks)

(iii) What is meant by packing fraction of a crystal?
Determine the packing fraction of a bcc crystal (2+3 marks)

(c) (i) Write down the translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ of the primitive cell of an fcc lattice in terms of its lattice constant 'a' (2 marks)

(ii) Use the above results to find the Miller indices of a (100) plane as referred to its primitive axes. (3 marks)

Question Two.

(a) (i) State the main features of ionic bonding in crystals. (3 marks)

(ii) Explain covalent bonding in crystals. Give at least one example. (3 marks)

(b) Derive an expression for the total lattice energy of a one -dimensional crystal formed as a line of 2N ions of alternating charge ± q at their equilibrium separation 'R_o'.The repulsive interaction may be assumed to be of the form **λexp(-r/ρ).**

(12 marks)

(c) A line of 2N ions of alternating charges (+/-) q have a repulsive potential energy of the form A/R^n, between nearest neighbours. Show that at equilibrium separation the potential energy,

$$U_{tot} = \frac{-2Nq^2 \ln 2}{R_0}\left(1-\frac{1}{n}\right).$$

[Given: Madelung constant = 2 ln2] (7 marks)

Question Three.

(a) State the following in crystal diffraction:

(i) Bragg's law.

(ii) Laue's condition (2 marks)

(b) Both x-rays and electrons can be used for crystal diffraction experiments. If the lattice constant of crystal is 1 Å, calculate:

(i) The energy- of x-ray photons that can be used in diffraction experiment.

(ii) The energy of an electron that can be diffracted by the crystal

[Given: photon energy $E = hc/\lambda$, Electron energy $E = h^2/(2m\lambda^2)$]

(4+ 4 marks)

(iii) State why electron beam cannot be used for study of bulk materials as compared to x-rays.

(2 marks)

(c) The geometric structure factor of a crystal is given below :

$S_G = \sum_{j=1}^{s} f_j \exp[-i2\pi(n_1 h + n_2 k + n_3 l)]$, where 's' is the number of atoms in the basis and n_1, n_2, n_3 are fractional coordinates. 'f' is the atomic form factor.

Show that diffraction can occur from planes in a diamond crystal only if

$$(h + k + l) = 4n$$

where n is any integer and h,k.l are the Miller indices. (8 marks)

(d) With the help of a diagram explain why a bcc lattice does not contain diffraction lines corresponding to (100) planes.

(5 marks)

Question Four

(a) (i) Define Fermi Energy. (2 marks)

(ii) Write down the Fermi - Dirac (F- D) distribution function for a system of fermions. (2 marks)

(iii) Compute the values of the F-D distribution function for the following cases at absolute zero temperature.

1. energy of the fermion $\epsilon >$ Fermi energy ϵ_F
2. energy of the fermion $\epsilon <$ Fermi energy ϵ_F (4 marks)

(iv) In a single sketch show how the Fermi function varies with energy at T = 0K and also at T > 0K and comment on the physical meaning of your observations. (5 marks)

(b) (i) Using the Schrödinger wave equation, show that the energy of a free electron is:

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$, where symbols have their usual, meanings. (4 marks)

(ii) Use the results in (i) above to show how the Fermi energy is related to the electron concentration, and hence derive an expression for the density of states of the electrons in a metal.

(8 marks)

Question Five.

(a) Explain how electrical conductivity of a pure semiconductor can be increased by:

(i) Thermal generation of carriers
(ii) Doping.

Give example where necessary (6 marks)

(b) With the help of appropriate energy band diagram; show that the density of electrons in the conduction band of a semiconductor is give by the expression:

$$2\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \exp\left[\frac{\varepsilon_F - \varepsilon_g}{kT}\right]$$

where symbols have their usual meanings. [Assume $(\epsilon - \epsilon_F) \ggg kT$]

Given: $\int_0^\infty \exp(-nx)x^{1/2}dx = \frac{1}{2n}\sqrt{\pi}$

(12 marks)

(c) A silicon sample is doped with $10^{17}cm^{-3}$arsenic atoms. All dopants are ionised.

1. What is the equilibrium hole concentration?
2. Where is the Fermi level relative to the centre of the band gap?

[intrinsic carrier concentration of silicon is $1.5x10^{10}cm^{-3}$]

(7 marks)

PHYSICAL CONSTANTS

Quantity	Symbol	Value
Angstrom unit	Å	1 Å = 10^{-8} cm = 10^{-10} m
Avogadro number	N	6.023×10^{23}/mol
Boltzmann constant	k	8.620×10^{-5} eV/K = 1.381×10^{-23} J/K
Electronic charge	q	1.602×10^{-19} C
Electron rest mass	m_o	9.109×10^{-31} kg
Electron volt	eV	1 eV = 1.602×10^{-19} J
Gas constant	R	1.987 cal/mole-K
Permeability of free space	μ_o	1.257×10^{-6} H/m
Permittivity of free space	ε_o	8.850×10^{-12} F/m
Planck constant	h	6.626×10^{-34} J-s
Proton rest mass	m_p	1.673×10^{-27} kg
$h/2\pi$	$\hbar$	1.054×10^{-34} J-s
Thermal voltage at 300 K	V_T	0.02586 V
Velocity of light in vacuum	c	2.998×10^{10} cm/s
Wavelength of 1-eV quantum	λ	1.24 μm