UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2006/2007

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

THIS PAPER CONTAINS FIVE QUESTIONS. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE.

(a) (i) Explain the difference between a **macrostate** and a **microstate** of a system of particles. (2 marks)

(ii) What is meant by **statistical weight** of a macrostate in a system of particles? (2 marks)

(iii) A classical system has 5 particles to be arranged in 3 energy levels. Using appropriate expression find the weights and hence the most probable distribution of these particles for the following cases:

1. The energy levels are non-degenerate.

2. The energy levels are doubly degenerate

(12 marks)

(b) (i) What is meant by Phase space? (2 marks)

(ii) Derive expressions for the volume element in phase space in terms of

1. Momentum p

2. Energy ϵ. (3 + 4 marks)

QUESTION TWO.

(a) Derive the Fermi-Dirac distribution function for a system of fermions,

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} + 1}$$, where symbols have their usual meanings

(12 marks)

(b) (i) Given that the density of states for fermions is:

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

where symbols have their usual meanings, show that the Fermi energy of a system of fermions:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

(8 marks)

(ii) Calculate the Fermi energy of a metal having density 8.5×10^2 kg m^{-3} and atomic weight 40. (5 marks)

QUESTION THREE.

(a) The Maxwell- Boltzmann distribution function for a system of classical particles is given by:

$$n_s = g_s e^{\alpha + \beta \varepsilon_s},$$

where the symbols have their usual meanings. Such a system has 2000 particles distributed in three non-degenerate energy levels having energies 1 unit and 2 units and 3 units each. The total energy is 2600 units. Use the above distribution function to obtain the values of α and β of this system and hence find its probable configuration.
Verify your answer numerically.

(15 marks)

(b) The differential form of Maxwell-Boltzmann distribution function in terms of the velocity v of the particles is given as:

$$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} v^2 dv$$

Use this expression to obtain:

(i) The mean velocity

(ii) The most probable velocity of the molecules of a classical gas.

[Note: see appendix for definite integrals]

(10 marks)

QUESTION FOUR.

(a) Use the Bose-Einstein distribution function of an assembly of identical non-interacting particles in thermal equilibrium to derive the Planck's radiation law for spectral distribution of energy radiated from a constant temperature enclosure.
(9 marks)

(b) Obtain an expression for the total energy per unit volume emitted from the enclosure at temperature T. (5 marks)

(c) (i) State briefly what is ***Bose_Einstein condensation.*** (3 marks)

(ii) The density of ideal gas consisting of particles having mass 6.65×10^{-27} kg is 1.17×10^{26} m^{-3}.

1. Calculate the Bose temperature T_B of the gas. (5 marks)

2. What fraction of the particles will be in the ground state at a temperature of $0.1T_B$. (3 marks)

Given: $N = 2.612V\left(\frac{2\pi mkT_B}{h^2}\right)^{3/2}$

QUESTION FIVE.

(a) Derive the partition function of a classical gas:

$$Z = \frac{v}{h^3}(2\pi mkT)^{3/2}$$

(8 marks)

(b) Show that the pressure of the classical gas:

$$P = NkT\frac{\partial \ln Z}{\partial V}$$

Hence derive the ideal gas equation $P\,V = N\,k\,T$ (10 marks)

(c) Calculate the translational partition function of an hydrogen molecule confined to a volume of 100 cm^3 at 300 K .

(7 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ ms^{-1}$
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	$1.38 \times 10^{-23}\ JK^{-1}$
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	$1.67 \times 10^{-27\,kg}$
Gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ JT^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Stefan constant	σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Atmospheric pressure		$1.01 \times 10^{5}\ Nm^{-2}$
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		$22.4\ l\ mol^{-1}$