UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2007/08

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Prove the law of sines using the cross product. Hint : Consider the area of a triangle formed by vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$. **(5 marks)**

(b) Let $\vec{A}$ be a vector in some arbitrary direction and $\hat{n}$ be a unit vector in some fixed direction. Show that $\vec{A} = (\vec{A} \bullet \hat{n})\hat{n} + (\hat{n} \times \vec{A}) \times \hat{n}$. **(5 marks)**

(c) Write down the acceleration in plane polar coordinates and state the meaning of each term. **(5 marks)**

(d) A projectile if fired at an angle θ with the horizontal over a flat surface.

(i) Show that the time it takes to reach the highest point is

$$t = \frac{\upsilon_0 \sin\theta}{g}.$$ **(2 marks)**

(ii) Show that the maximum height reached is

$$h = \frac{\upsilon_0^{\,2} \sin^2\theta}{2g}.$$ **(4 marks)**

(iii) Show that the range is given by

$$R = \frac{\upsilon_0^{\,2} \sin 2\theta}{g}.$$ **(4 marks)**

QUESTION 2

(a) What horizontal force F is to be applied to the cart shown in figure 1 so that the blocks of masses m_1 and m_2 remain stationary with respect to the cart. Assume all surfaces are frictionless. The answer must be in terms of m_1, m_2, m_3 and g. **(4 marks)**

(b) The two blocks shown in figure 2 are connected with a string of negligible mass. If the system is released from rest. Find how far block m_1 slides in time t. All surfaces are frictionless. **(6 marks)**

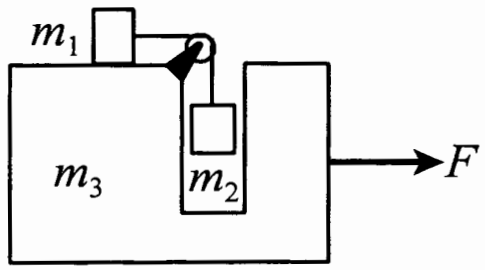

Figure 1.

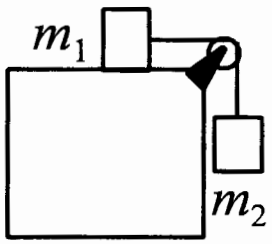

Figure 2.

(c) A mass m is connected by two strings of length l to a vertical exle revolving with angular velocity ω. Each of the strings make an angle of 60° with the axle, as shown in Figure 3. Gravity is directed downward.

(i) Drawn a clear appropriately resolved force diagram for the mass m. **(3 marks)**

(ii) Find the tensions T_1 and T_2 in the strings in terms of g, l, m and ω. **(12 marks)**

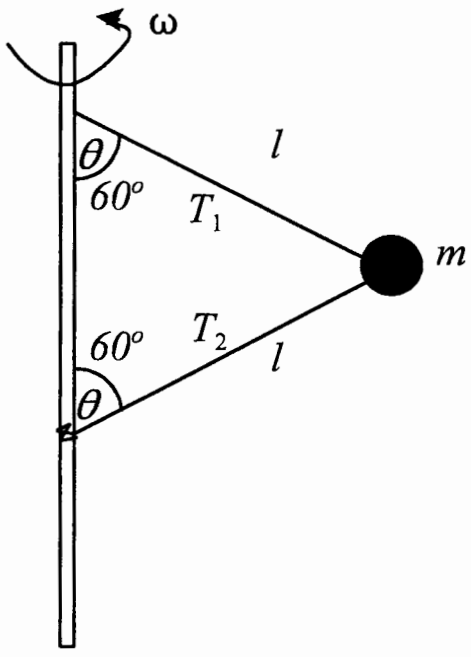

Figure 3.

QUESTION 3

(a) To reduce the force of impact with the ground, animals instinctively prolong the time of collision by lowering their centre of mass by a distance s when they fall to the ground.

(i) Show that the time for the collision is given by $t = \frac{2s}{\upsilon_o}$, where υ_0 is the incident velocity. **(3 marks)**

(ii) Show that if the animal free falls from rest over a height h the force of impact will be $F = mg\frac{h}{s}$, where m is the mass of the animal and g is the gravitational acceleration. **(3 marks)**

(iii) A construction worker of mass 80 kg accidentally slips from a height of 7 m and falls rigidly on the ground. His center of mass only changes by 2 cm during impact with the ground. The bone contact area in each ankle is 5 cm^2. If the compressive strength of the human bone is about 1.8×10^8 Pa, determine whether the person is likely to have fractures in his ankles. **(3 marks)**

(b) An empty rail car of mass M_0 starts from rest under an applied force F. At the same time, maize begins to fill the car at a steady rate $dm/dt = b$ from a hopper at rest along the rail track (Figure 4). Find the velocity when a mass, m, of maize has been transferred to the rail car. The problem can be solved in only two steps, but use the *mass and momentum transport* method. Apply your solution to the case when $M_0 = 500$ kg, $b = 20$ kg/s and $F = 100$ N to find the velocity at time $t = 10$ s. **(12 marks)**

(c) Water of density ρ shoots out of a fire hydrant having nozzle diameter D with nozzle speed V_0. What is the force on the hydrant in terms of ρ, D and V_0? **(4 marks)**

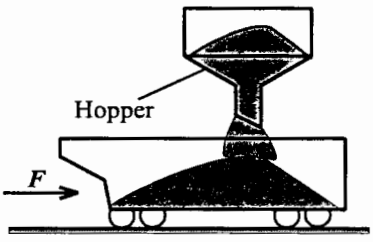

Figure 4.

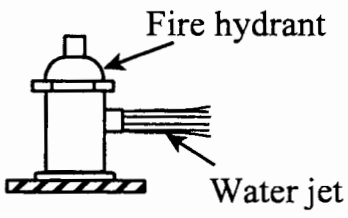

Figure 5.

QUESTION 4

(a) Derive an expression for the work-energy theorem. **(7 marks)**

(b) A block of mass m slides from rest from the top of a frictionless sphere of radius R (see Figure 4.) How far below the top does it lose contact with the sphere? The sphere does not move. **(8 marks)**

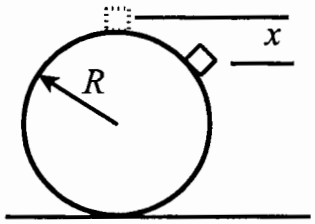

Figure 4.

(c) A body of mass m moves along the positive x-axis in the potential well given by

$$U = Bx + \frac{A}{x} + C,$$ where A, B, and C are positive constants.

(i) What is the force on the body? **(2 marks)**
(ii) Determine the equilibrium point(s) for the body. **(2 marks)**
(iii) Determine the stability of the equilibrium point(s). **(3 marks)**
(iv) What is the angular frequency of small oscillations about the equilibrium point? **(3 marks)**

QUESTION 5

(a) Show that when a particle has potential energy U and kinetic energy K of the form

$$U = \frac{1}{2} Aq^2 + C, \text{ and}$$

$$K = \frac{1}{2} B\dot{q}^2,$$

where q is a variable appropriate to the problem, and A, B, and C are constants, the particle oscillates harmonically with angular frequency $\omega = \sqrt{A/B}$. **(6 marks)**

(b) A conical pendulum has a string of length l attached at point B making an angle α with the vertical (see Figure 5).

Find the angular momentum for a conical pendulum at

(i) point A which is the centre of the horizontal circle of radius r in which the mass m moves, and **(6 marks)**

(ii) point B where the pendulum string is fixed and make a diagram that illustrates the angular momentum at this point. **(8 marks)**

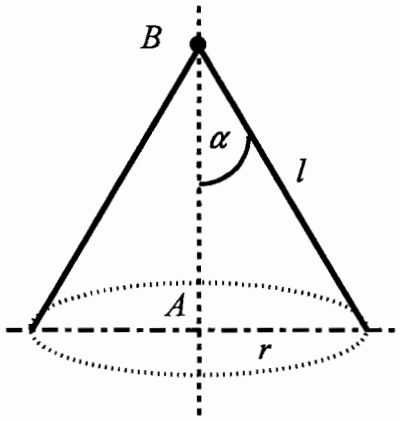

Figure 5.

(c) Show that the torque $\vec{\tau}$ and angular momentum $\vec{L}$ are related by the following equation:

$$\vec{\tau} = \frac{d\vec{L}}{dt}.$$ **(5 marks)**