UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2007/08

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR

QUESTION 1

(a) A lifesaver sees a dog being washed down a river with a velocity u = 5 km/hr, which is the speed of the water current down stream. The lifesaver sees the dog when it is 120 m upstream. The boat can obtain an instantaneous maximum speed v = 8 km/hr. The width of the river is L= 100 m. Refer to Figure 1.

(i) Suppose the lifesaver drives the boat with its velocity v so that the resultant velocity of the boat is directly across the river to point *C*. (That means the lifesaver has to drive the boat at an angle θ upstream from the line *AC*). Determine from the times taken by the dog and the boat to reach point *C* whether the dog can be saved. **(5 marks)**

(ii) Now suppose that the lifesaver drives the boat with its velocity *v* directly along the line *AC*. (In this case the boat cannot go along the line *AC* but will be swept by the water down stream and will follow the path *AD* which is at an angle ϕ with the line *AC*.) Determine from the times taken by the dog and the boat to reach point *D* whether the dog will be saved. **(6 marks)**

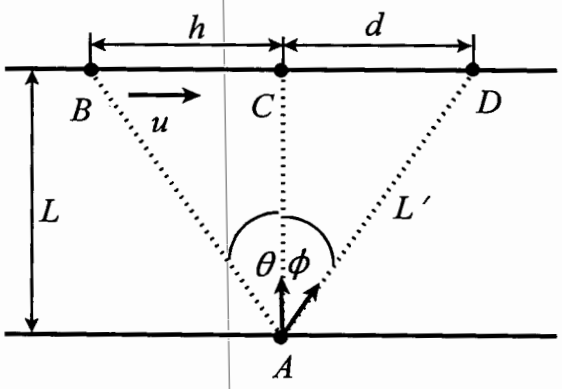

Figure 1.

(b) (i) Use a clear diagram as an aid to show the polar unit vectors $\hat{r}$ and $\hat{\theta}$ in terms of Cartesian unit vectors $\hat{i}$ and $\hat{j}$. **(4 marks)**

(ii) Show that $\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$. **(5 marks)**

(iii) Show that $\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{\theta}$. **(5 marks)**

QUESTION 2

(a) A particle of mass $m_1 = m$ and a second of mass $m_2 = 2m$ are connected by a light inextensible string of length $2L$. The string passes through a frictionless ring at a fixed height. Mass m_2 hangs at rest a distance of $L/2$ below the ring while mass m_1 moves in a horizontal circle. See Figure 2.

(i) Find the angular velocity ω of the mass m_1 in terms of g and L. **(9 marks)**

(ii) What is the radius of its motion? **(4 marks)**

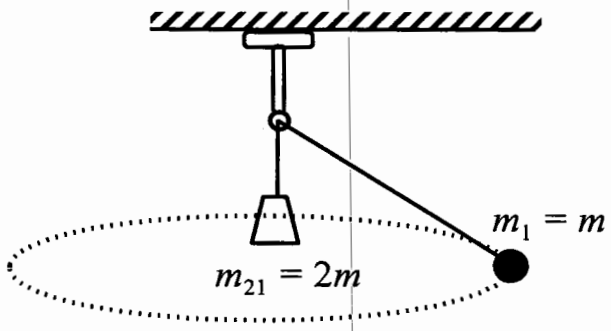

Figure 2.

(b) A rain drop of mass m falls from rest under gravity. Its motion is resisted by air resistance whose magnitude is $F_R = kv$, where k is a constant.

(i) Make a force diagram and find an expression for the acceleration of the rain drop. **(2 marks)**

(ii) Find the velocity of the rain drop as a function of time. **(7 marks)**

(iii) What is the terminal velocity of the rain drop? **(3 marks)**

QUESTION 3

(a) Find the centre of mass of a cone of mass M, radius R and height $h = 2R$. The volume of a cone is base area divided by 3 multiplied by the height. **(9 marks)**

(b) Water shoots out of a fire hose with a horizontal velocity of $\upsilon_0 = 30$ m/s. The diameter of the nozzle is $D = 3$ cm. The rate at which water leaves the nozzle is $dm/dt = 1.5$ kg/s. Find the force required to hold the hose steadily. **(5 marks)**

(c) In a high jump competition a 70 kg man jumps to a height of 2.2 m. What impulse does the man get from the ground? **(5 marks)**

(d) A science teacher throws a tennis ball at a student with a velocity υ and the student catches the ball without feeling pain. The teacher then throws an object ten times as heavy as the tennis ball. For the following cases determine the relationship (greater than, equal to, or less than) between the force of impact of the tennis ball and that of the heavy object. Assume the collision time is the same in all cases.

(i) The heavy object is thrown at the same speed as the tennis ball. **(2 marks)**
(ii) The heavy object is thrown with the same momentum as the tennis ball. **(2 marks)**
(iii) The heavy object is thrown with the same kinetic energy as the tennis ball. **(2 marks)**

QUESTION 4

(a) A particle move from the highest point of a smooth sphere of radius R with an initial velocity u. See Figure 3.

(i) Determine the normal force on the particle while it is still on the sphere in terms of u, R, g and θ. Hint: Use the work-energy theorem. **(9 marks)**

(ii) Find an expression for the angle θ at which the particle leaves the sphere and evaluate the angle θ for $u = \sqrt{\frac{Rg}{2}}$. **(5 marks)**

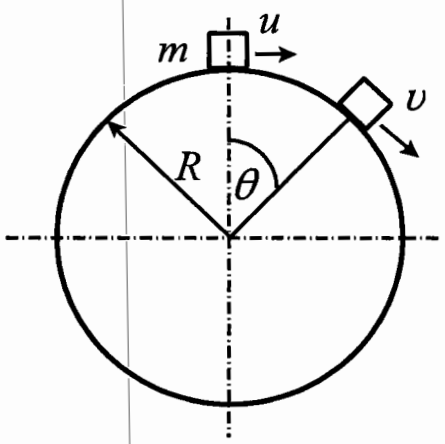

Figure 3.

(b) A potential energy function on a particle of mass m is given by $U = 3Ax^3 - 72Bx$, where A and B are positive constants with appropriate units.

(i) Find the force acting on the body. **(3 marks)**
(ii) Find the equilibrium points. **(2 marks)**
(iii) Determine the stability of the equilibrium points. **(3 marks)**
(iv) Find the frequency of small oscillations about the equilibrium points. **(3 marks)**

QUESTION 5

(a) Find the moment of inertia of a stick of length L about an axis through the origin ($x = 10$). The length density of the stick varies as $\lambda = \frac{\lambda_0 x}{L}$, where λ_0 is a constant. **(6 marks)**

(b) A block slides in the positive x direction and is slowed down by a frictional force $f = \mu N$, where N is the normal force and μ is the coefficient of kinetic friction (See Figure 4). Find the angular momentum and the torque about an axis through point P at $y = l$. **(6 marks)**

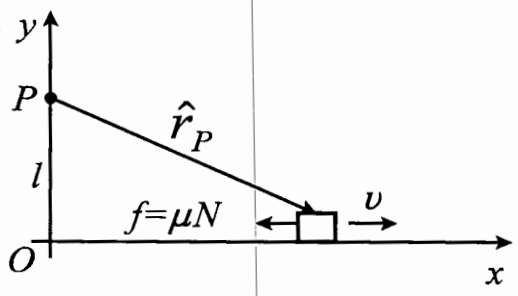

Figure 4.

(c) (i) Derive with the aid of diagrams an equation of motion for a physical pendulum and show that it oscillates. **(7 marks)**

(ii) Determine the angular frequency of the oscillation in terms of the radius of gyration k, the distance of the pivot from the centre of mass l, and the gravitational acceleration g. **(4 marks)**

(iii) Using the result from (ii), find the angular velocity of a simple pendulum. **(2 marks)**