UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2007/08

TITLE O F PAPER: MODERN PHYSICS & WAVE OPTICS

COURSE NUMBER: P231

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

THE LAST PAGE CONTAINS FORMULAE AND CONSTANTS THAT MAY BE USEFUL IN SOME PROBLEMS

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR

QUESTION 1.

(a) Two narrow, parallel slits separated by d = 0.250 mm are illuminated by green light (λ = 546.1 nm). The interference pattern is observed on a screen 1.20 m away from the plane of the slits.

(i) Generally what should be the path difference for to beams to interfere constructively? **(2 marks)**

(ii) In general what should be the path difference for two beams to interfere destructively? **(2 marks)**

(iii) In the stated problem, find the distance between bright fringes. **(4 marks)**

(iv) In the stated problem, find the distance between dark fringes. **(4 marks)**

(v) First compare the results for (iii) and (iv). In the equations used in (iii) and (iv) an approximation has been made. To what maximum fringe number from the axis is this approximations valid for say the bright fringes? **(5 marks)**

(b) A screen is placed a distance L = 50 cm from a single slit of width a, which is illuminated with light of wavelength λ = 690 nm. The distance between the first and third minima in the diffraction pattern is 3 mm.

(i) What is this kind of diffraction called and what is the condition for minima at small angles (θ)? **(3 marks)**

(ii) What is the width of the slit in the given problem? **(5 marks)**

QUESTION 2

(a) Determine the minimum distance between two point sources that the human eye can distinguish at the near point (25 cm), assuming that the pupil diameter is 2 mm, and that a wavelength of 500 nm is used. **(5 marks)**

(b) (i) With the aid of diagrams fully discuss polarisation by reflection. **(10 marks)**

(ii) Assume the polarising block has a refractive index $n_2 = n$ and that the unpolarised light is incident from air with refractive index $n_1 = 1$, find a relationship between refractive index of the block n and the angle of incidence (θ_B) for 100 % linear polarisation of the reflected beam. **(4 marks)**

(c) What minimum thickness of calcite will make a half-wave plate for sodium light of wavelength $\lambda = 589.3$ nm. The indices of refraction for the O and the E rays are $n_O = 1.6584$ and $n_E = 1.4864$, respectively. **(6 marks)**

QUESTION 3

(a) Assuming the human body and a tungsten filament both emit like blackbodies,

(i) calculate the peak wavelengths of the human skin at a temperature of 37 °C, and a tungsten incandescent light bulb at a temperature of 3000 K. **(3 marks)**

(ii) Calculate the intensity of per unit area, $I(\lambda, T)$ at the temperatures given in (i) for the peak sensitivity of the human eye $\lambda = 550$ nm, and use this information to discuss why you are able to clearly see the light of the tungsten light bulb but not that of the human being on a dark night? **(5 marks)**

(b) Discuss the photoelectric effect with relevant diagrams and equations and state whether it supports the particle or wave nature of light. **(13 marks)**

(c) A 0.00160 nm X-ray Compton scatters from a free electron. For which photon scattering angle does the recoiling electron have the same kinetic energy as the scattered photon? **(4 marks)**

QUESTION 4

(a) State the postulates Bohr made to derive the model for a hydrogen atom. Include relevant equations. **(6 marks)**

(b) Show that the radius of orbit of the electron in the n^{th} Bohr orbit in hydrogen is given by

$$r_n = \frac{n^2\hbar^2}{m_e k_e e^2}.$$

where the symbols have their usual meaning. Justify all you steps. **(6 marks)**

(c) First explain what a hydrogen-like ion is and generalise the equation in (b) for any hydrogen-like ion. **(4 marks)**

(d) State the limitation of the Bohr model for the hydrogen atom and explain how it was resolved by subsequent work. In particular, discuss the role of the wave function and give a short outline of how solving the Schrodinger equation for hydrogen-like atoms introduces three quantum numbers which address these limitations. **(9 marks)**

QUESTION 5

(a) Starting with the equation for the decay rate of N nuclei

$$\frac{dN}{dt} = -\lambda N,$$

where λ is the decay constant, show that the half-life of a radioactive substance is given by

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$

(6 marks)

(b) A radioactive sample contains 3.50 μg of pure carbon 11 $\left({}^{11}_{6}C\right)$, which has a half-life of 20.4 min.

(i) What is the number of carbon 11 nuclei in the sample? **(1 mark)**

(ii) Find the activity of the sample in becquerels initially. **(2 marks)**

(iii) Find the activity after exactly eight hours. **(1 mark)**

(iv) What is the number of remaining nuclei after eight hours. **(1 mark)**

(c) Consider a neutron with an energy above 1 Mev traveling through matter. Discuss how such a neutron can be captured and write down the equation for neutron capture and explain it. First discuss with the aid of calculations whether or not neutrons of the given kinetic energy can be easily captured. **(10 marks)**

(d) Sketch the potential energy function for two deuterons as a function of separation and explain it. **(4 marks)**

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

Avogadro's number N_A 6.022 x 10^{23} paricles/mol
Stefan-Boltzmann constant σ = 5.669 6 x 10^{-8} W/$(m^2K^2)^3$
Boltzmann's constant, k_B = 1.3801 x 10^{-23} J/K
Bohr magneton, μ_B = 9.27 x 10-24 J/T
Bohr radius $a_0 = \hbar^2/(m_e k_e e^2)$ = 0.0529 nm
Rydberg constant R_H = 1.097 373 2 x 10^7 m^{-1}
Speed of light in vacuum, c = 2.997 924 58 x 10^8 m/s
Planck's constant, h = 6.626 075 x 10^{-34} J·s
$\hbar$ = 1.054 572 x 10^{-34} J·s
hc = 1.986 447 x 10^{-25} J·m
$2\pi hc^2$ = 3.741 859 x 10^{-15} J·m^2s^{-1}
Compton wavelength $\lambda_c = h/(m_e c)$ = 0.002 43 nm
mass of an electron, m_e = 9.109 389 7 x 10^{-31} kg
mass of a proton, m_p = 1.672 623 x 10^{-27} kg
mass of a neutron, m_n = 1.674 928 6 x 10^{-27} kg
Coulomb constant, k_e = 8.987 551 787 x 10^9 Nm^2/C^2
electron charge, e = 1.602 177 33 x 10^{-19} C
1 atomic mass unit = 1 amu = 1 u = 1.660 540 2 x 10^{-27} kg = 931.494 MeV
1 eV = 1.602 177 33 x 10^{-19} J
1Ci = 3.7 x 10^{10} Beq
$T_{1/2}(^{14}C)$ = 5730 years

$$\frac{N(^{14}C)}{N(^{12}C)} = 1.3x10^{-12}$$

Hydrogen (^1_1H) mass 1.007 825

Helium (^4_2He) mass = 4.002 602 u

Nitrogen ($^{15}_7N$) mass = 15.000 109 u

Oxygen ($^{15}_8O$)mass = 15.003 065 u

Fluorine ($^{19}_9F$) mass = 18.998 404 u

$$\lambda_{max} = \frac{hc}{4.965k_BT}$$

$$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda k_B T} - 1)}$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$\sin\theta \approx \tan\theta$, for θ less than 3°