

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2007/2008

TITLE OF PAPER : THERMODYNAMICS

COURSE NUMBER : P242

TIME ALLOWED : THREE HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

**MARKS FOR DIFFERENT SECTIONS ARE
SHOWN IN THE RIGHT-HAND MARGIN.**

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

**DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE
INVIGILATOR.**

INFORMATION

For a monatomic gas: $\gamma = \frac{5}{3}$ and $C_v = \frac{3R}{2}$; $C_p = \frac{5R}{2}$

For a diatomic gas: $\gamma = \frac{7}{5}$

Universal gas constant = $8.31 \text{ mol}^{-1}\text{K}^{-1}$

Specific heat of water = $4190 \text{ J kg}^{-1}\text{K}^{-1}$

Latent heat of vaporisation of water = $2.256 \times 10^6 \text{ J kg}^{-1}$

Stefan-Boltzmann constant = $5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4}$

QUESTION 1

- (a) Consider an ideal gas which is caused to move through a p-V cycle ABCA. A to B is an isochoric process, B to C is an isothermal expansion, and C to A is an isobaric process.
- (i) For each leg of the cycle, taking the gas as the system, determine whether each of the following quantities is positive, negative or zero: change in temperature, heat absorbed, work done by gas, change in internal energy, change in entropy. (10 marks)
- (ii) For the whole cycle — A to B to C to A — answer the same questions. (3 marks)
- (iii) Operated as stated, is this the cycle of a heat engine or refrigerator? Give reasons for your answer. (2 marks)
- (b) A heat reservoir at 373 K is used to evaporate 0.2 kg of water originally at 313 K.
- (i) How much energy must flow from the reservoir to do this? (6 marks)
- (ii) What is the change entropy of the water? (4 marks)

QUESTION 2

- (a) The overall volume expansion ratio of a Carnot cycle shown in Fig. 2.1 is 15. The temperature limits of the cycle are 21°C (step C-D) and 260°C (step A-B)

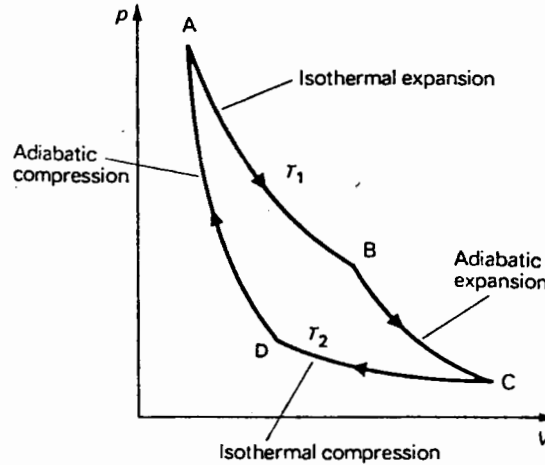


Fig. 2.1

Determine the volume ratios of the isothermal and adiabatic processes.

(10 marks)

[Take $\gamma = C_p/C_v = 1.4$]

- (b) Imagine a Carnot engine which takes 5000 J heat each cycle from the high-temperature reservoir at 300 K and gives out 3500 J to the low-temperature reservoir.
- (i) Calculate the temperature of the low-temperature reservoir. (4 marks)
- (ii) What is the thermal efficiency of the cycle? (2 marks)
- (c) A real engine takes in 210 kJ of heat at 700 K and expels 140 kJ at 400 K.
- (i) If the real engine is replaced by a Carnot engine working between the same two temperatures with an input of 20 kJ, how much heat would be expelled and how much work would it do? (5 marks)
- (ii) What would be the co-efficient of performance of the Carnot engine if it were used as a heat pump between the same two temperatures? (4 marks)

QUESTION 3

- (a) A heating element is embedded in a ceramic rod 0.05 m in length and its emissivity is 0.50. If the ceramic rod is to be maintained at 737°C with a power of 1 kW supplied to the element, what should the radius of the rod be?

[Assume that the surroundings are at 0 K and that negligible heat losses occur from the ends of the rod]. (5 marks)

- (b) A house without an insulation in the ceiling loses 200 J of heat per second per square metre of the ceiling when the inside temperature is heated to 40°C and the outside temperature is 0°C. The owner decides to add glass wool insulation [thermal conductivity $0.042 \text{ Js}^{-1}\text{m}^{-1}(\text{C}^\circ)^{-1}$] to the ceiling. How thick should the insulation be in order to cut the heat loss by 80%? (6 marks)

- (c) A sphere of radius R, held at temperature T_0 , radiates as a black body. Determine an expression for the power flux at a distance $d \gg R$ from the centre of the sphere. (5 marks)

- (d) The surface temperature of a spherical mass of molten metal (the source) is T_1 and its radius is r_1 . It is surrounded by a spherical shell of inside radius r_1 , outside radius r_2 and thermal conductivity k as shown in Fig. ???. The outside of the shell is at temperature T_2 . Assuming k is constant, show that the radial rate of flow of heat in a substance between the two concentric spheres is given by

$$\frac{dQ}{dt} = \frac{(T_1 - T_2)4\pi kr_1 r_2}{(r_2 - r_1)} \quad (9 \text{ marks})$$

QUESTION 4

- (a) The change in internal energy of an ideal gas undergoing an adiabatic process is given by the following equation:

$$\Delta U = C_v \Delta T, \text{ where } T = \text{temperature};$$

C_v is the heat capacity at constant volume

Show that $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

and that the work done

$$W = \frac{RT_1}{(\gamma - 1)} \left[1 - \left(\frac{T_2}{T_1} \right) \right]$$

(7 marks)

- (b) Two thousand moles of a monatomic ideal gas is taken through the following cyclic process (1) An isobaric expansion from 2 m^3 to 4.6 m^3 at a pressure of $4 \times 10^6 \text{ Pa}$; (2) An isochoric decrease in pressure from $4 \times 10^6 \text{ Pa}$ to $1 \times 10^6 \text{ Pa}$; (3) An adiabatic compression back to the initial state.
- (i) Show the p-V diagram for this cyclic process; (2 marks)
- (ii) What is the work done during each step of the cycle and the net heat for the cycle? (9 marks)
- (iii) What is the heat exchanged during each step of the cycle and the net heat for the cycle? (7 marks)

QUESTION 5

- (a) Show that the pressure of an ideal gas is related to the density of the gas and the mean square speed of the gas molecules as indicated in the equation below.

$$p = \frac{1}{3} \rho \overline{v^2} \quad (15 \text{ marks})$$

- (b) Assume that the speed of sound in an ideal gas is the same as the root-mean-square speed of the gas molecules and show that the speed of sound in an ideal gas depends on temperature.

(5 marks)

- (c) Consider an ideal monatomic gas whose molecular mass and root-mean-square molecular speed are $4 \times 10^{-3} \text{ kg mol}^{-1}$ and 1360 ms^{-1} respectively.

Find the temperature of the gas.

(5 marks)