UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2007/08

TITLE OF THE PAPER: **COMPUTATIONAL PHYSICS -I**

COURSE NUMBER : **P262**

TIME ALLOWED : **THREE HOURS**

SECTION A: ONE HOUR.
SECTION B: TWO HOURS.

INSTRUCTIONS:

SECTION A **: THIS IS WRITTEN PART ON YOUR ANSWER BOOK. CARRIES A TOTAL OF 30 MARKS.**

SECTION B **: THIS IS A PRACTICAL PART AND CARRIES A TOTAL OF 70 MARKS.**

ANSWER **ANY THREE** QUESTIONS FROM **SECTION A** AND **ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS FOUR PAGES, INCLUDING THIS PAGE.
DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Part) **Time: One Hour**

Q.1. In an R, L, C circuit, frequency f is given by the equation [10]

$$f = \sqrt{\frac{1}{LC} - \frac{R^2}{4C^2}}$$

For L=2 x 10^{14} H and R=2.2 kΩ, use Maple command **"fsolve"** to calculate C for frequencies 100Hz to 500Hz in steps of 50Hz.

Q.2. Given a set of data points (x_i ,y_i) for i=1..10, use Maple command **"sum"** to calculate [10]

$$\sum_{i=1}^{N} x_i \quad , \quad \sum_{i=1}^{N} y_i \quad , \quad \sum_{i=1}^{N} x_i y_i \quad , \quad \sum_{i=1}^{N} x_i^2$$

$$a = \frac{-\sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i y_i + \sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \quad , \quad b = \frac{N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}$$

where *N=10* refers to total number of data points.
The parameters *a* and *b* are least squares fit parameters for a straight line *y= a + b x*.

Q.3. (a) Write Maple commands to produce a set of data points (x_i, y_i) for *i=1..10* using the equation [6]

$$y = 2.3x^2 + 4.5x - 1.5$$

for $1 \le x \le 5$.

(b) Write Maple commands to plot the data (x_i ,y_i) for i=1..10 and the curve corresponding to the equation above in the same graph. [4]

Q.4. (a) Following is the integral given symbolically:

$$\int_{0.001}^{z} x^5 \frac{\sin(\pi x)}{(e^x - 1)} dx$$

Write Maple commands to

(i) to enter the integral symbolically. [2]
(ii) to evaluate it analytically. [2]
(iii) to calculate the integral numerically for z =1,2,3,4,5 using a **" for"** loop, and print all the values. [6]

SECTION B
(Practical Part) Time: Two Hours.

Q.5. Van der Waal's equation for an **imperfect** gas leads to the following equation for volume V for a given pressure P and temperature T

$$V^3 - \left(b + \frac{RT}{P}\right)V^2 - \frac{a}{P}V - \frac{ab}{P} = 0$$

$R=8.3149 \times 10^3$ J kg^{-1} $mole^{-1}$ K^{-1} .
Constants a and b are related to critical pressure P_c and temperature T_c .
They are given by the relation:

$$a = \frac{27R^2T_c^2}{64P_c} \quad , \quad b = \frac{RT_c}{8P_c}$$

For CO_2 , T_c = 304.26 K and $P_c = 7.40 \times 10^6$ Pa.
Use Maple to

(i) Calculate a and b . [4]

(ii) Plot volume V for an ideal gas using the equation $V = \frac{RT}{P}$ [4]
at temperature 300^0K from pressures 5×10^6 Pa to 10×10^6 Pa .

(iii) Use the calculated values of a and b to calculate volume V using **fsolve** command for the imperfect gas at temperature 300^0 K and at pressures 5×10^6 Pa to 10×10^6 Pa with an increment of 1×10^6 Pa. [15]

(iv) Plot on the same graph V vs P for the ideal and imperfect gas. [2]

(v) For each value of V and P calculate the speed of sound ***u*** in an imperfect gas given by the expression [10]

$$u = \left\{\frac{C_p}{\mu C_v}\left[\frac{RT}{(v-b)^2} - \frac{2a}{v^3}\right]\right\}^{1/2}$$

where for CO_2 , μ=molecular weight 44.01 and C_p / C_v = 1.3.

Print P vs u.

Q.6.. A young kid of mass m=30 kg dives into a 6 m deep lake from a low cliff, entering the water perpendicular to the surface at a speed of 10m/s. Surface co-ordinate of the water is x=0. The diver did not know swimming and hence only the buoyant force of the water brought him to the surface of water in 20 s. Since the velocity involved is low, the drag force due to water is given by F_{drag}= - k v where v is the velocity. Assume the specific gravity of the diver to be μ=0.95.

The net external force is the sum of drag force (F_{drag}= - k v) , the weight (-mg) and the buoyant force (mg/μ). Here g=9.8 , is the acceleration due to gravity. This leads to the equation of motion (Newton's second law Force=mass x acceleration) as given below:

$$m\frac{d^2x(t)}{dt^2} = -k\frac{dx(t)}{dt} - mg + \frac{mg}{sg}.$$

Here velocity $v = \frac{dx(t)}{dt}$.

We note that at t=0, x(0)=0 and $v(0) = \left.\frac{dx(t)}{dt}\right|_{t=0} = -10$.

Use Maple to

(i) Solve the differential equation to obtain an expression for x(t). with the given initial conditions. [10]

(ii) From the solution of x(t) obtain an expression for v(t). [5]

(iii) The variable x(t) at time t=20 s corresponds to the surface of water, i.e. x(t)=0 at t=20. Use this information to obtain the value of k. ***Note:You may need to use the command "fsolve" with the range for k=0..100***. [5]

(iv) Use the value of k determined in (iii) to find the time taken to reach the velocity v(t)=0. Hence determine the maximum depth reached by the diver. [10]

(v) Plot x(t) vs t and v(t) vs t for t=0..25. [5]

@@@END OF EXAMINATION@@@